

1. NOWAK HEPATITIS B MODEL

In a pioneer and effective effort to model chronic HBV infection dynamics and its treatment with the reverse transcriptase inhibitor lamivudine, Nowak et al. employed the following phenomenological model

$$\begin{aligned}\frac{dx}{dt} &= -\beta SI - \beta_F SF \\ \frac{dy}{dt} &= \beta SI + \beta_F SF - \gamma I \\ \frac{dv}{dt} &= \gamma I - \alpha F\end{aligned}$$

Where x, y, z are number of uninfected (susceptible) hepatocyte, infected hepatocyte, and free virion respectively. Uninfected hepatocytes are assumed to be produced at a constant rate, λ , to maintain tissue homeostasis in the face of hepatocyte turnover, described by the linear term λx , where d is the per-capita death rate. A healthy liver maintains $\frac{\lambda}{d}$ cells as its homeostatic set-point. However, during infection, healthy (uninfected) hepatocytes are assumed to become infected at rate βvx , where β is the mass action rate constant describing the infection process. Infected hepatocytes are killed by immune cells at rate γy and produce free virions at rate $k y$, where k is the so-called burst constant. Free virions are cleared by lymphatic and other mechanisms at rate αv , where μ is constant. Here the amount of virus lost (which is βvx if it takes exactly one virus to infect a hepatocyte) due to infection process is assumed as negligible and thus is omitted from the virus equation.

Look at the paper: <http://www.pnas.org/content/pnas/93/9/4398.full.pdf>

Try to analyze long term dynamics of this model. What does the reproduction number in the paper mean?