1. Gyllenberg-Webb model for tumor growth

Can be found on page 38 of (2016 book Introduction to Mathematical Oncology by Y. Kuang, J. D. Nagy and S. E. Eikenberry, published by CRC Press).

Gyllenberg and Webb explore the consequences of two simple observations from real tumors: (1) actively proliferating cells can enter a quiescent state where they stop dividing; and (2) quiescence tends to be more common in large compared to small tumors. Quiescence appears to be a cellular response to stress, like hypoxia, nutrient limitation, or increased hydrostatic pressure. Such stresses typically increase, often nonlinearly, with tumor size. It can be reversible or irreversible. Gyllenberg and Webb propose a simple, but rather general, model of the transition into and out of quiescence. Let P(t) and Q(t) be the number of proliferative and quiescent cells, respectively. Define N(t) = P(t) + Q(t). Then the Gyllenberg-Webb model takes the following form:

$$\frac{dP}{dt} = (\beta - \mu_p - r_0(N))P + r_i(N)Q$$
$$\frac{dQ}{dt} = r_0(N)P - (r_i(N) + \mu_q)Q$$

with initial conditions $P(0) = P_0 > 0$ and $Q(0) = Q_0 > 0$. Here β is the reproduction rate of proliferating cells, and μ_p and μ_q are the death rates of proliferating and quiescent cells, respectively. The transition rates r_0 and r_i are known functions of N that can be chosen to best fit reality. In this project, you investigate the above tumor model above by looking at the evolution in time of the number of quiescent and proliferating cells. Further, you can consider several extensions of the model to include three compartments or time delay