Sample 1

1. Question 1

a) Nonlinear Univariate Discrete Deterministic

b) $N^* = 1 + N^* - \frac{(N^*)^2}{4}$
$1 = \frac{(N^*)^2}{4}$
$4 = (N^*)^2$
$N^* = \pm 2$

c) $f(N) = 1 + N - \frac{N^2}{4}$
$f'(N) = 1 - \frac{N}{2}$
$f'(2) = 1 - \frac{2}{2} = 0$
$f'(-2) = 1 - \frac{-2}{2} = 2$
Since $|f'(2)| < 1$ the fixed point 2 is stable
Since $|f'(-2)| > 1$ the fixed point -2 is unstable

2. Question 3

a) Linear Multivariate Discrete Deterministic

b) $x_1(t+1) = \frac{1}{2}x_1(t) - x_2(t)$
$x_2(t+1) = ax_1(t) - \frac{1}{2}x_2(t)$

$x_1^* = \frac{1}{2}x_1^* - x_2^*$
$x_2^* = ax_1^* - \frac{1}{2}x_2^*$

$x_1^* = -2x_2^*$
$x_2^* = a * (-2x_2^*) - \frac{1}{2}x_2^*$
$x_2^*(\frac{3}{2} + 2a) = 0$
Therefore if $a \neq \frac{-3}{4}$ then the fixed point is (0,0)

c) $Trace(A) = 0$, $Det(A) = \frac{-1}{4} + a$
$\lambda^2 + a - \frac{1}{4} = 0$
$\lambda^2 = \frac{1}{4} - a$
$\lambda = \pm\sqrt{\frac{1}{4} - a}$
$\sqrt{\frac{1}{4} - a} < 1$ implies $-\frac{3}{4} < a < \frac{1}{4}$
Therefore the fixed point (0,0) is stable when $-\frac{3}{4} < a < \frac{1}{4}$

d)In the case $a = \frac{-3}{4}$ from solving the fixed point we find that $x_1^* = -2x_2^*$ is a solution, which is a line of equilibria. These equilibria will not be stable for two reasons:
1) Part c) says $a > \frac{-3}{4}$
2) For any given fixed point, the initial condition as close to it as possible that is still on the line $x_1^* = -2x_2^*$ will not move since it is also a fixed point. Therefor the original fixed point is unstable.

3. Question 4

a) Nonlinear Univariate Continuous Deterministic

b)$0 = 1 - (x^*)^2$
$x^* = \pm 1$

c)$f(x) = 1 - x^2$
$f'(x) = -2x$
$f'(-1) = 2 > 0$, therefore the fixed point -1 is unstable
$f'(1) = -2 < 0$, therefore the fixed point 1 is stable

d) $\frac{dx}{(1-x)(1+x)} = dt$
Using partial fractions, we get:
$\frac{1}{(1-x)(1+x)} = \frac{a}{1-x} + \frac{b}{1+x}$
$1 = a(1+x) + b(1-x)$
let x=1, then $1 = 2a$ therefore $a = \frac{1}{2}$
let x=-1, then $1 = 2b$ therefore $b = \frac{1}{2}$
Therefore we have:
$\frac{dx}{2(1-x)} + \frac{dx}{2(x+1)} = dt$
$\frac{dx}{(1-x)} + \frac{dx}{(x+1)} = 2dt$
$\int \frac{1}{1-x} dx + \int \frac{1}{x+1} dx = 2 \int 1 dt$
$-ln(x-1) + ln(x+1) = 2t + C_1$
$ln(\frac{x+1}{x-1}) = 2t + C_1$
$\frac{x+1}{x-1} = C_2 e^{2t}$
$x + 1 = C_2 e^{2t}(x-1)$
$1 + C_2 e^{2t} = (C_2 e^{2t} - 1)x$
$x(t) = \frac{1+C_2 e^{2t}}{C_2 e^{2t} - 1}$
at $t = 0$, $x(t) = x(0) = \frac{1+C_2}{C_2 - 1}$
$x(0)(C_2 - 1) = 1 + C_2$

$C_2(x(0)-1) = 1 + x(0)$
$C_2 = \frac{1+x(0)}{x(0)-1}$
$x(t) = \frac{1+(\frac{1+x(0)}{x(0)-1})e^{2t}}{(\frac{1+x(0)}{x(0)-1})e^{2t}-1}$

Sample 2

4. Question 1

a) Nonlinear Univariate Discrete Deterministic

b) $N^* = N^*(0.3 + (1-\alpha)N^*)$
$N^* = 0$ or
$1 = (0.3 + (1-\alpha)N^*)$
$0.7 = (1-\alpha)N^*$
$N^* = \frac{0.7}{1-\alpha}$, assuming $\alpha \neq 1$

c) $f(N) = N(0.3 + (1-\alpha)N)$
$f'(N) = 0.3 + 2(1-\alpha)N$
$f'(0) = 0.3$
$f'(\frac{0.7}{1-\alpha}) = 0.3 + 2 * 0.7 = 1.7$
Since $|f'(0)| < 1$ the fixed point 2 is stable
Since $|f'(\frac{0.7}{1-\alpha})| > 1$ the fixed point $\frac{0.7}{1-\alpha}$ is unstable

5. Question 4

a) linear Univariate Continuous Deterministic

b)$0 = 2x$
$x^* = 0$

c)$f(x) = 2x$
$f'(x) = 2$
$f'(0) = 2 > 0$, therefore the fixed point 0 is unstable

d) $\frac{dx}{x} = 2dt$
$\frac{dx}{x)} = 2dt$
$\int \frac{1}{x}dx = 2\int 1dt$
$ln(x) = 2t + C_1$
$x = C_2e^{2t}$
$x(t) = C_2e^{2t}$

$x(0) = 5 = C_2$
$x(t) = 5e^{2t}$