

$$\begin{pmatrix} 0 & f \\ s_J & s_A \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

To find eigenvalues:

$$\begin{pmatrix} 0 & f \\ s_J & s_A \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} - \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -\lambda & f \\ s_J & s_A - \lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

Determining the value of the eigenvalues:

$$-\lambda(s_A - \lambda) - fs_J = 0$$

$$\lambda^2 - \lambda s_A - fs_J = 0$$

$$\lambda = \frac{s_A \pm \sqrt{s_A^2 + 4fs_J}}{2}$$

Using $\lambda_1 = \lambda_+$:

$$-\lambda v_1 + f v_2 = 0$$

$$v_1 = \frac{f v_2}{\lambda}$$

$$v_1 = \frac{f v_2}{\frac{s_A + \sqrt{s_A^2 + 4fs_J}}{2}}$$

$$v_1 = \frac{2f v_2}{s_A + \sqrt{s_A^2 + 4fs_J}}$$

$$v_1 = \frac{2f v_2}{s_A + \sqrt{s_A^2 + 4fs_J}} \frac{s_A - \sqrt{s_A^2 + 4fs_J}}{s_A - \sqrt{s_A^2 + 4fs_J}}$$

$$v_1 = \frac{2f v_2 (s_A - \sqrt{s_A^2 + 4fs_J})}{s_A^2 - s_A^2 - 4fs_J}$$

$$v_1 = \frac{v_2 (s_A - \sqrt{s_A^2 + 4fs_J})}{-2s_J}$$

$$v_1 = \frac{\sqrt{s_A^2 + 4fs_J} - s_A v_2}{2s_J}$$

Second row of expanded form:

$$s_J v_1 + s_A v_2 - \lambda v_2 = 0$$

$$v_1 = \frac{\lambda v_2 - s_A v_2}{s_J}$$

$$v_1 = \frac{\frac{s_A + \sqrt{s_A^2 + 4fs_J}}{2} v_2 - s_A v_2}{s_J}$$

$$v_1 = \frac{\sqrt{s_A^2 + 4fs_J} - s_A v_2}{2s_J}$$

You can see that v_1 is the same in both situations. (You don't need to do that twice, I'm just demonstrating that they are the same). Now, set $v_2 = 1$ and see that $v_1 = \frac{\sqrt{s_A^2 + 4fs_J} - s_A}{2s_J}$. Therefore, the eigenvector associated with λ_+ is $\left(\frac{\sqrt{s_A^2 + 4fs_J} - s_A}{2s_J}, 1 \right)$.