

MATH 3MB3 FALL 2018 HOMEWORK 2  
Due Monday October 15 at 11:59PM

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Format your report using some form of word processing software (Word, Latex, OpenOffice, ...), export it to a PDF file and submit it via email to:

- Alexandra Bushby, bushbya@mcmaster.ca if your last name starts with A-G or if you are submitting using R
- Robert White, whitere@mcmaster.ca if your last name starts with a H-Z, or you plan on using Python, and you don't plan on using R

together with a file containing the code you used for your computer simulations.

Make sure that your email has the proper subject line and information from the outline.

### QUESTION 1

Lets let  $A$  be the number of young adults in a population and  $E$  be the number of elderly people in the population. We assume that the birth rate for young adults is  $b$ , and the birth rate for elderly is 0. Furthermore we assume that survival rates for adults and elderly are  $s_A, s_E$  respectively. It is further assumed that if a young adult survives for the year they magically become elderly. Recall that all rates are always between 0 and 1.

- a) Find the eigenvalues
- b) Find the eigenvectors (do not simplify the complicated one)
- c) Use a) and b) to setup the explicit solution given initial values  $A(0) = A_0$  and  $E(0) = E_0$ . Do NOT solve for  $C_1, C_2$

For the rest of question 1) use computer software and set  $s_A = \frac{1}{10}$ ,  $s_E = \frac{1}{20}$ ,  $b = \frac{1}{4}$ ,  $A(0) = 50$ ,  $E(0) = 20$ :

- d) What is the explicit solution now?

- e) Calculate  $A(10)$  and  $E(10)$
- f) Describe what you expect to happen to  $A$  and  $E$  over time. Justify

using b), c), d), e).

g) Plot A and E as functions of time (at least till time 10), and describe their long term dynamics

#### 1. QUESTION 2

Given the system  $x' = x^2 - 2x - 3$ .

- a) Classify the model
- b) Find the fixed points
- c) Find the stability of the fixed points
- d) Find an explicit solution of the model

For the following use computer software:

- e) Run three numerical experiments with initial conditions of ( $N(0) = -5$ ,  $N(0) = 1$ ,  $N(0) = 10$ ) and use the explicit formula with  $t = 1, 10, 100$  to determine the long term dynamics of each initial condition.
- f) Do your results in part e) agree with part b), c)? Explain!