MATH 3MB3 FALL 2018 HOMEWORK 3
Due Monday Nov 5 at 11:59PM

Format your report using some form of word processing software (Word, Latex, OpenOffice, ...), export it to a PDF file and submit it via email to:

- Alexandra Bushby, bushbya@mcmaster.ca if your last name starts with A-G or if you are submitting using R
- Robert White, whitere@mcmaster.ca if your last name starts with a H-Z, or you plan on using Python, and you don't plan on using R

together with a file containing the code you used for your computer simulations.
Make sure that your email has the proper subject line and information from the outline.

Question 1

This question involves and explores the relationship between discrete and continuous models. First we start with a continuous model
$\frac{dx}{dt} = rx$
Then we discretize this equation using the Backward Euler method:
$\frac{dx}{dt} \approx \frac{x(t)-x(t-h)}{h}$
where h is a parameter, one obtains:
$X(t+h) = \frac{X(t)}{1-rh}$
We now denote it as capital X just to differentiate from the x in the continuous model we had before. Assume that x(0)=X(0)=a
a) Show how the third equation is obtained from the first two equations.
b) Find the solution x(t) of the first equation
c) Find an explicit expression for X(t) from the last equation, assuming that $t = nh$ with n being an integer.

Now set $r = 1.6$ and $a = 2$. Answer the following questions/parts with the aid of computer software. The domain of time that we are interested in is t=0 to t=6.
d) Graph x(t). On the same plot, graph X(t) for h=0.01 and h=0.001. Comment on your observations. Give the values of all three quantities at t=6.

e) Graph the absolute global truncation error $E(t) = |x(t) - X(t)|$, again for both h=0.01 and h=0.001. What does E(t) represent? What is the value of E(6) for h=0.01? What is E(6) for h=0.001? Explain the difference between both values and speculate where the error comes from.
f) Graph the relative global truncation error $e(t) = \frac{|x(t)-X(t)|}{x(t)}$, again for both h=0.01 and h=0.001. What do you observe?

QUESTION 2

Consider the following matrix equation:
$\frac{d\vec{x}}{dt} = \begin{bmatrix} \frac{-1}{2} & 4 \\ -4 & \frac{-1}{2} \end{bmatrix} \vec{x}$
Where $\vec{x} = (x_1(t), x_2(t))$. Let A be the matrix above.
a) Classify this model
b) Find the fixed points of this model
c) Determine the stability of each fixed point. Furthermore describe what you expect the solution to look like near the fixed point.
d) Find the explicit solution $\vec{x(t)}$
Assume that $\vec{x(0)} = (-3, 3)$. e) Using a computer, write down the explicit solution for this initial condition, and graph it as well.