project_example

October 24, 2017

0.1 Question

Many of Canada's lakes contain populations of fish that are predators of smaller-bodied fish. Under a wide variety of natural conditions these predator-prey systems are stable. Humans interact with these systems in at least two ways: (1) by fishing for larger-bodied predatory fish and (2) artificially enhancing predatory fish stocks. Here we analyze a simple model of predator-prey interactions to ask how fishing and stocking influence the stability of this system.

0.2 Model construction

0.2.1 Variables

$V(t)$ and $P(t)$ are the numbers of prey (V for victims) and predators in year t.

0.2.2 Assumptions and parameters

I construct my model in discrete time with a time step of one year. This assumption is reasonable given that most fish spawn once a year. We assume that in the absence of predators, prey increase in abundance according to a linear model. Similarly, in the absence of prey, predators die according to a linear model.

We assume that prey are captured by predators according to the mass-action principle. That is, any particular predator and any particular prey have equal chance of encountering each other, regardless of the distance between them. Therefore, predators capture prey at a rate proportional to $V(t)P(t)$ with proportionality constant c. We assume that predators convert captured prey into new predators with efficiency, e. That is, if x prey are captured in one time step, ex new predators will appear next step.

We assume that predators are fished by a fishery that has a quota of h fish per year. Furthermore, we assume that the fishery never under- or overshoots its quota. Finally, we also assume that each year m predator fish are added to the lake by natural resource managers.

0.2.3 Model equations in discrete time

These assumptions lead to the following set of equations for the dynamics of the predator-prey system:

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\begin{aligned}
V(t+1) &= V(t) + aV(t) - cV(t)P(t) \\
P(t+1) &= P(t) + ecV(t)P(t) - fP(t) + m - h
\end{aligned}
$$

where a is the growth rate of prey in the absence of predators ([time^{-1}]), c is the per-predator-prey-encounter rate of prey capture ([time^{-1} predator^{-1}], e is the efficiency of predators at converting captured prey into resources ([predator prey^{-1}]), f is the per-predator mortality rate ([time^{-1}]), m is the rate of stocking ([predator time^{-1}]) and h is the rate of fishing harvest ([predator time^{-1}]).

0.3 Proposed analysis

I will research the literature on fresh water fisheries in Canada to propose default values for all parameters other than m and h. Given these default values, I will attempt to analytically find the values of $m-h$ that lead to stability of the system. Having gained knowledge from this analysis, I will modify the above model with t[he aim of making it more realistic. For example, mortality is expected to occur continuously throughout the a year, and so a potential adjustment to this model could be adding seasonal dynamics. Another possible modification could be to relax the mass-action assumption. Then I will analyze this modified model using a combination of stability analysis and numerical solutions.

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