MATH 3MB3 FALL 2018 HOMEWORK 4
Due Friday Nov 30 at 11:59PM

Format your report using some form of word processing software (Word, Latex, OpenOffice, ...), export it to a PDF file and submit it via email to:

- Alexandra Bushby, bushbya@mcmaster.ca if your last name starts with A-G or if you are submitting using R
- Robert White, whitere@mcmaster.ca if your last name starts with a H-Z, or you plan on using Python, and you don't plan on using R

together with a file containing the code you used for your computer simulations.

Make sure that your email has the proper subject line and information from the outline. Furthermore pictures of written work is not a form of word processing.

QUESTION 1

Consider the following model similar to the good/bad crops example done in class:

$N(t+1) = (1+r^3)N(t)$

Here N is the population size, t is time and r is a random variable that is normally distributed with mean $\mu = -0.1$ and standard deviation $\delta = 0.2$. This means $r \approx N(-0.1, 0.2)$. Also assume that $N(0) = 100$. The goal is now to investigate what happens to the population over time.

a) Classify this model

b) Find the fixed points

c) Produce a histogram of 10000 samples of the random parameter r^3. Compute the mean and standard deviation of the generated histogram (This is a computer question)

d) What do you expect to happen to the population over time? Support your answer using c).

e) Run a single trial/simulation of the model up to time $t = 200$. Graph N as a function of t. (This is a computer question).

f) Now run 1000 trials up to t=50. Produce a histogram and again find the mean and standard deviation. (This is a computer question).

g) Verify your numerical results by comparing them to your conclusion

from d).

QUESTION 2

An employee at a large corporation enjoys a yearly wage of 120000 dollars. She spends half of this money and deposits the other half in her savings account at the bank. Due to fluctuations in the world economy, the banks yearly interest rate varies from year to year. In a given year, chances are 1 in 4 that the interest rate is 0.5%, 1 in 2 it is 1%, and 1 in 4 it is 1.5%. Additionally, the employee gets a yearly bonus that she deposits directly into her savings account. The bonus is an amount that is normally distributed with mean 8000 dollars and standard deviation 1600 dollars.

You may assume that the employees entire wage is paid out on January 1st, while the bonus she receives is paid out on December 31st. Let M(t) be the amount of money in the savings account and t is time (in years). Assume M(0) = 100000 so that the employee starts off with 100000 dollars. We are interested in what happens to M over time.

a) Create a model for M by writing down a recursive formula for M(t).
b) Classify your model.
Use computer software to answer the following questions.
c) Run a single trial of the model up to t = 30. Show a plot of M as a function of t.
d) Now run 10000 trials of the model up to t = 30. Display a histogram of M at t = 30. Compute the mean and standard deviation of this distribution.
e) Run 10000 more trials of the model and calculate how many years it will most likely take for the employee to cross the million dollar mark on her savings account (i.e., balance exceeding 1000000). Hint: add an if-statement in your time-stepping loop to calculate when one million dollar is.