

MATH3MB3 Exam Review

1.

$$\begin{aligned}S(t+1) &= S(t) + m[N - S(t)] - S(t)[1 - e^{-\beta I(t)}] \\I(t+1) &= I(t) + S(t)[1 - e^{-\beta I(t)}] - [m + \gamma]I(t)\end{aligned}$$

- a) NMDD
- b) $(S_*, I_*) = (N, 0)$
- c) Stable if $|1 - m| < 1$ and $|1 + \beta N - (m + \gamma)| < 1$

2.

$$N(t+1) - N(t) = R_{\max} N \frac{1 - N(t)}{N_{\max}}$$

- a) NUDD
- b) $N_* = 0$ and $N_* = N_{\max}$
- c) $N_* = 0$: stable if $|1 + R_{\max}| < 1$ and $N_* = N_{\max}$: stable if $|1 - R_{\max}| < 1$

3.

$$N(t+2) = N(t+1) + 3N(t)$$

- a) LUDD
- b) $N_* = 0$
- c) Unstable
- d) $N(t) = \frac{\sqrt{13}}{13} \left[\frac{1+\sqrt{13}}{2} \right]^t - \frac{\sqrt{13}}{13} \left[\frac{1-\sqrt{13}}{2} \right]^t$

4.

$$\begin{aligned}\frac{dS(t)}{dt} &= -\mathcal{R}_0 SI \\ \frac{dI(t)}{dt} &= \mathcal{R}_0 SI - I\end{aligned}$$

- a) NMCD
- b) $(S_*, I_*) = (S, 0)$, where $S \in \mathbb{R}$
- c) unstable if $S > \frac{1}{\mathcal{R}_0}$

5.

$$\begin{aligned}P(t+1) &= \gamma\alpha\sigma P(t) + \sigma\beta S(t) \\S(t+1) &= \gamma\sigma(1-\alpha)P(t)\end{aligned}$$

- a) LMDD
- b) $(P_*, S_*) = (0, 0)$
- c) If $\gamma\sigma^2\beta(1-\alpha) > 0$ and $\gamma\alpha\sigma < 0$ stable, otherwise unstable.

6.

$$\frac{dx}{dt} = a - bx$$

- a) LUCD
- b) $x_* = \frac{a}{b}$
- c) stable if $b < 0$
- d) $x(t) = \frac{a}{b} + ce^{-bt}$

7.

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

- a) NUCD
- b) $N_* = 0$ and $N_* = K$
- c) $N_* = 0$: stable if $r < 0$ and $N_* = K$: stable if $r > 0$
- d) *don't try to solve for this equation. Sorry about that.*

8.

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 + 2x_2 \\ \frac{dx_2}{dt} &= 3x_1 + 2x_2\end{aligned}$$

- a) LMCD
- b) $(x_1^*, x_2^*) = (0, 0)$
- c) saddle, unstable.