$\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ $\qquad \frac{\ln(n)}{n} > \frac{1}{n}$ for $n \geq 3$

$$\therefore \quad \sum_{n=3}^{\infty} \frac{\ln(n)}{n} > \sum_{n=3}^{\infty} \frac{1}{n}$$

$$\therefore \quad 0 + \frac{\ln(2)}{2} + \sum_{n=3}^{\infty} \frac{\ln(n)}{n} > 0 + \frac{\ln(2)}{2} + \sum_{n=3}^{\infty} \frac{1}{n}$$

$$\therefore \quad \sum_{n=1}^{\infty} \frac{\ln(n)}{n} > 0 + \frac{\ln(2)}{2} + \sum_{n=3}^{\infty} \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{3n^2+n}{\sqrt[3]{n^7+1}} \qquad \sum_{n=1}^{\infty} \frac{4^n+5}{7^n-8}$$

Alternating Series

An <u>alternating series</u> is a series whose terms are alternating positive and negative.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

Alternating Series Test

Consider $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$, $b_n > 0$, satisfying

i) $b_n \geq b_{n+1}$ (monotonic decreasing)

ii) $\lim_{n\to\infty} b_n = 0$

Then $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ converges

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = (-1)\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

$b_n = 1/n$ b_n is decreasing and $\lim_{n\to\infty} b_n = 0$

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges by AST

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges

Determine if the following converge.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4n}{5n+6} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{n}$$

Alternating Series Estimate

If the alternating series converges,

then the remainder $|R_n| = |s - s_n| \le b_{n+1}$

Find the sum of $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ correct to 3

decimal places, do not simplify.