The Principle of Mathematical Induction

Let P_n be a statement involving the positive integer n. Assume the following

a) P_1 is true
b) If P_k is true, then P_{k+1} is true

Then by P.M.I P_n is true for all positive integers.

Example: Use P.M.I to show that
$$1+2+3+\dots+n = \frac{n(n+1)}{2}.$$

let P_n be $1+2+\dots+n = n(n+1)/2$.

a) Check P_1. $1(1+1)/2 = 1$ $\therefore$ P_1 is true

b) Assume that P_k is true, i.e
$$1+2+\dots+k = \frac{k(k+1)}{2}$$

To try to get to the statement P_{k+1}

we add what is missing to both sides $(k+1)$

to get
$$1+2+\dots+k+k+1 = k(k+1)/2 + (k+1)$$
$$= (k+1)\left(\frac{k}{2}+1\right)$$
$$= (k+1)\left(\frac{k+2}{2}\right)$$
$$= \frac{(k+1)((k+1)+1)}{2}$$

$\therefore$ by P.M.I P_n is true for all n.