Power Series

A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \dots$$

where x is a variable and the c_n's are constants called the coefficients of the series.

A series of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \dots$$

is called a power series in $(x-a)$ or a power series centered at a.

Ex For what values of x is the series $\sum_{n=0}^{\infty} n!\, x^n$ convergent?

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \left|\frac{x^{n+1}}{x^n}\right|$$

$$= \lim_{n\to\infty} (n+1)|x| = \infty$$

$\therefore$ The series is divergent everywhere except for $x=0$.

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$$

Theorem

For a given series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities

a) The series converges only for $x=a$

b) The series converges for all x

c) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$

In this case R is called the <u>radius of convergence</u>. In case a) $R=0$, In case b) $R=\infty$.

The <u>interval of convergence</u> is the interval that consists of all values of x for which the series converges.

Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n\sqrt{n+1}}$ check endpoints.