$$\int_{n+1}^{N+1} f(x)\,dx \le \sum_{k=n+1}^{N} a_k \le \int_{n}^{N} f(x)\,dx$$

let $N \to \infty$ then

$$\int_{n+1}^{\infty} f(x)\,dx \le \sum_{k=n+1}^{\infty} a_k \le \int_{n}^{\infty} f(x)\,dx$$

$$\text{//} \quad \le \quad S - S_n \quad \le \quad \text{//}$$

$N+1$ since $N-n+1$ terms

Absolute Convergence

If $\sum |a_n|$ is convergent, the $\sum a_n$ is said to be <u>absolutely convergent.</u>

If $\sum a_n$ converges, but is not absolutely convergent, then it is <u>conditionally convergent.</u>

Note an absolutely convergent series must be convergent since

$\sum a_n \leq \sum |a_n|$ then

$$0 \leq \sum a_n + \sum |a_n| \leq 2 \sum |a_n|$$

Are the following conditionally convergent, absolutely convergent, or divergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad\qquad \sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

Ratio Test

Let $\sum a_n$ be a series such that

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L.$$

a) If $L < 1$ then $\sum a_n$ is absolutely convergent

b) If $L > 1$ then $\sum a_n$ is divergent

c) If $L = 1$, the ratio test tells us nothing.

Ex $\quad \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{e^n}$

The Root Test

Let $\sum a_n$ be a series such that

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$$

a) If $L < 1$ then $\sum a_n$ is absolutely convergent

b) If $L > 1$ then $\sum a_n$ is divergent

c) If $L = 1$ then the root test is inconclusive.

Ex $$\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+4}\right)^n$$