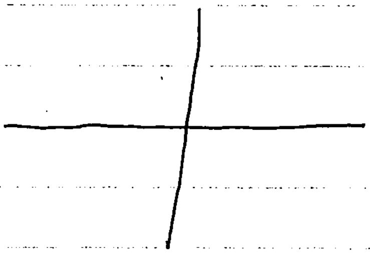
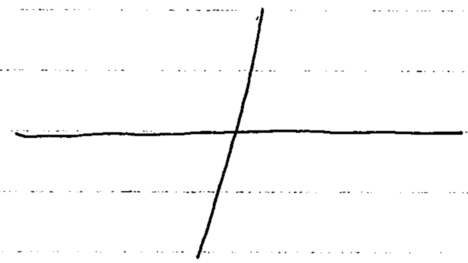
Sequences

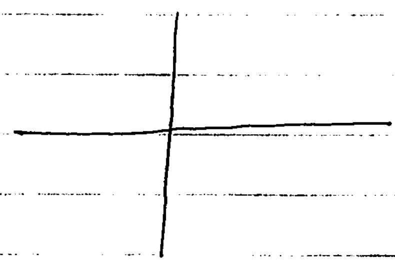
Sketch and state the domain

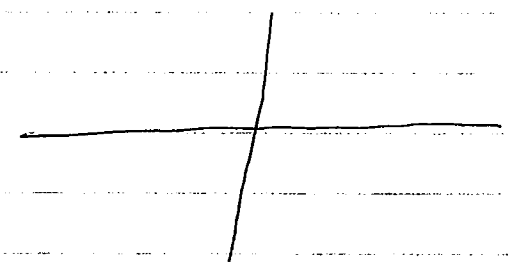
a) $f(x) = x^2$ b) $g(x) = x/(x+1)$





Sketch the above if the domain is restricted to the Natural Numbers





Notation

a) $a_n = n^2$

or

$\{n^2\}_1^\infty$

or

$\{n^2\}$

b) $b_n = n/(n+1)$

or

$\{n/(n+1)\}_1^\infty$

or

$\{n/(n+1)\}$

Definition:

An <u>infinite sequence</u> is a function whose domain is the set of positive integers.

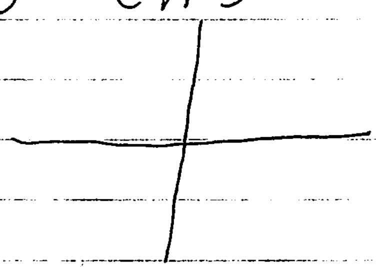
Examples write the first five terms of each sequence:

a) $\dfrac{n^2+2}{n^2+n+2}$ b) $\{\sqrt{n-3}\}_3^\infty$ $\cancel{\{\cos n\pi\}}$

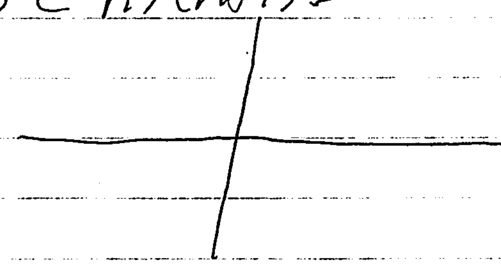
c) $\{\cos(n\pi)\}$

How do the following graphs of two sequences differ?

a) $\{n^2\}$



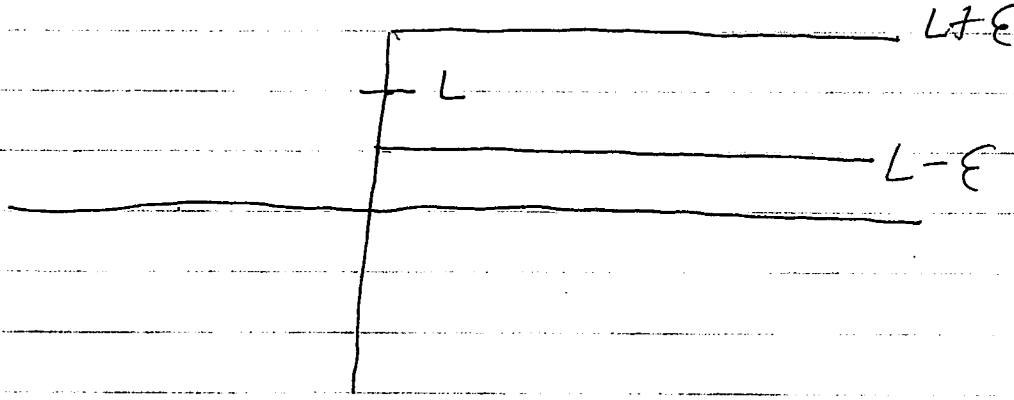
b $\{n/(n+1)\}$



$\lim_{n\to\infty} a_n = L$. provided the values of a_n get closer and closer to L as $n \to \infty$.

We say that the sequence $\{a_n\}$ converges to L, and $\{a_n\}$ is called a convergent sequence.

Note: What this means graphically is that only a finite number of elements of $\{a_n\}$ may be outside the "lane" determined by $(L-\varepsilon, L+\varepsilon)$ for any value of ε.



If there does not exist such L, then $\{a_n\}$ diverges, and is called a divergent sequence.

Determine whether the following sequences have limits:

a) $1/2^n$ b) $(n^2+2)/(2n^2+3n+7)$

c) $(-1)^n$ d) $(-1)^n/n$

e) $(n^2+1)/(n-1)$

Note in the case of e) we write "diverges to ∞".

Limit Theorems. If $\lim_{n\to\infty} a_n = A$, $\lim_{n\to\infty} b_n = B$

then 1. $\lim_{n\to\infty} (ka_n) =$

2. $\lim_{n\to\infty} (a_n \pm b_n) =$

3. $\lim_{n\to\infty} (a_n b_n) =$

4. Assuming $B \neq 0$, $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) =$

Sequences can also be given recursively.

Fibonacci Sequence

$a_1 = 1$ $a_2 = 1$ $a_n = a_{n-1} + a_{n-2}$ $n \geq 3$

To find the limit of this sequence

we need more tools.

Limit Squeeze Theorem

If $a_n \le b_n \le c_n$ for $n \ge N$, and

$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n$ then $\lim_{n\to\infty} b_n = L$.

Application $\{n!/n^n\}$

Monotonic Sequences:

$\{a_n\}$ is Monotonic increasing if $a_{n+1} > a_n$ for all $n \ge 1$

$\{a_n\}$ is Monotonic decreasing if $a_{n+1} < a_n$ for all $n \ge 1$

Bounded Sequences:

U is an upper bound of $\{a_n\}$ iff $a_n \le U$ for all $n \ge 1$

V is a lower bound of $\{a_n\}$ iff $a_n \ge V$ for all $n \ge 1$

$\{a_n\}$ is a bounded sequence if it has both an upper and lower bound.

To show a lower bound, show $a_n - V \geq 0$
" " an upper bound, show $U - a_n \geq 0$
" " monotonic increasing $a_{n+1} - a_n \geq 0$
" " " decreasing $a_n - a_{n+1} \geq 0$

If a sequence is monotonic increasing and is bounded above, then it is convergent.

Similarly if a sequence is bounded below and monotic decreasing, then it is convergent.

Example $a_1 = \sqrt{2}$
$a_{n+1} = \sqrt{2 + a_n}$

First show bounded.