Divergence Test

Similar to what we did with integrals,

if $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

However note that $\lim_{n\to\infty} a_n = 0$ does not guarantee convergence.

Example $\sum_{i=1}^{n} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \ldots$

Series

Let $S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \dots + a_n$

which is the n^{th} <u>partial sum</u>.

If $\lim_{n \to \infty} S_n = s$ exists, then we say that the series $\sum_{i=1}^{n} a_n$ is <u>convergent</u> and it's <u>sum</u> is s.

If the limit does not exist or is not finite the the series $\sum_{i=1}^{n} a_n$ <u>diverges</u>.

Let a be a non-zero real number, and r be a real number, then

$\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$ and converges to $a/(1-r)$.

Example: The series $\sum_{n=1}^{\infty} \frac{4^n}{5^n}$ converges to what?