Is $a_n = \frac{n}{n^2+1}$ monotonic?

If so how would you show this?

let $S_n = \sum_{i=1}^{n} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \left(\frac{1}{i} - \frac{1}{i+1}\right)$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{n} - \frac{1}{n+1}$

$= 1 - \frac{1}{n+1}$

Does this series converge?

Based on the above what is

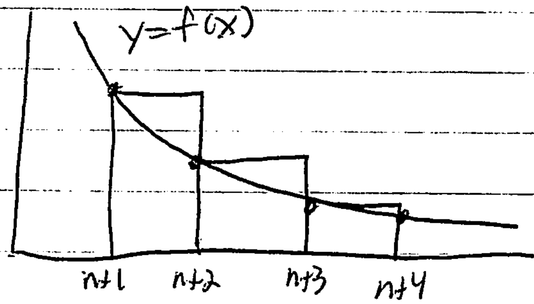
$$\sum_{n=1}^{\infty} \left(\frac{1}{n(n+1)} + \frac{1}{2^n}\right)$$

Integral Test.

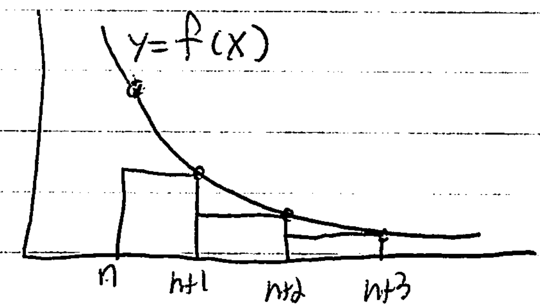
Let $f(x)$ be a continuous, positive, decreasing function defined for $x \geq 1$,

Let $a_n = f(n)$, for $n = 1, 2, \ldots$

Then the series $\sum_{n=1}^{\infty} a_n$ converges iff $\int_1^{\infty} f(x)\,dx$ converges.



$$\sum_{K=n+1}^{N} a_K > \int_{n+1}^{N+1} f(x)\,dx$$



$$\int_n^N f(x)\,dx > \sum_{k=n+1}^{N} a_K$$

furthermore $\quad s_n + \int_{n+1}^{\infty} f(x)\,dx \leq \sum_{k=1}^{\infty} a_K \leq s_n + \int_n^{\infty} f(x)\,dx$

Examples

Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges or diverges.

Determine if the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges or diverges.