Applications of Seperable Equations

A tank contains 20kg of salt dissolved in 5000L of water

Brine containing 0.03 kg of salt per liter enters at 25L/min.

The solution is thoorougly mixed and drains at a rate of 25L/min.

How much salt is in the system after 30 min?

$y(t)$ = amount of salt at time t. $\quad y(0)=20$.

$\frac{dy}{dt}$ = (rate in) − (rate out)

$$= .03 \cdot 25 - \frac{y(t)}{5000} \cdot 25$$

$$= .75 - \frac{y(t)}{200} = \frac{150 - y(t)}{200}$$

$$\int \frac{dy}{150-y(t)} = \int \frac{1}{200}\,dt \Rightarrow -\ln|150-y(t)| = \frac{t}{200} + C_1$$

$$\ln|150-y(t)| = \frac{-t}{200} + C_2 \qquad y(0) = 150 - C_3 = 20$$

$$C_3 = 130$$

$$150 - y(t) = e^{(-t/200 + C_2)} \qquad y(t) = 150 - e^{-t/200} \cdot 130$$

$$y(t) = 150 - e^{-t/200} C_3 \qquad y(30) = 150 - 30 e^{-30/200}$$

Population Growth

$P(t) = P_0 e^{kt}$ where $P(0) = P_0$ (the initial condition)

Given $P(0) = 2560$

$P(10) = 3560$ find k, then find $P(30)$.

$P(0) = 2560$ $\quad P(10) = 3560 = 2560 e^{k \cdot 10}$

$$\ln\left(\frac{3560}{2560}\right) = 10k$$

$$k = \frac{\ln\left(\frac{3560}{2560}\right)}{10}$$

$$P(30) = 2560 e^{\frac{\ln\left(\frac{3560}{2560}\right)}{10} \cdot 30}$$

$$= 2560 \left(\frac{3560}{2560}\right)^3$$

Radioactive Decay

$m(t) = m_0 e^{kt}$

The <u>half-life</u> of a radioactive substance is the amount it takes for half of the quantity to decay.

$$\tfrac{1}{2} m_0 = m_0 e^{kt}$$
$$\tfrac{1}{2} = e^{kt}$$

$$\ln(1/2) = kt$$
$$k = \frac{\ln(1/2)}{t} = \frac{-\ln(2)}{t}$$

Ex. If the half-life of an element is 150 years, find the mass after t years, if $m(0) = 100$.

$$m(t) = 100\, e^{-\frac{\ln(2)\, t}{150}}.$$

$$m(1000) = 100\, e^{-\frac{\ln(2) \cdot 1000}{150}}$$

When will the mass become 30 grams?

$$30 = 100\, e^{-\frac{\ln(2) \cdot t}{150}}$$

$$\ln\left(\frac{30}{100}\right) = \frac{-\ln(2)\, t}{150} \qquad \frac{-150 \ln\left(\frac{30}{100}\right)}{\ln(2)} = t$$