Find the area of the surface generated by rotating the curve $y = e^x$, $0 \le x \le 1$ around/about the x-axis.

$y = e^x \qquad dy/dx = e^x$

$$I = 2\pi \int_0^1 y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\pi \int_0^1 e^x \sqrt{1+e^{2x}}\,dx$$

let $u = e^x \qquad du = e^x\,dx$

$$I = 2\pi \int_1^e \sqrt{1+u^2}\,du \qquad \text{let } u = \tan\theta \qquad du = \sec^2\theta\,d\theta$$

$$I = 2\pi \int_{u=1}^{u=e} \sqrt{1+\tan^2\theta}\,\sec^2\theta\,d\theta = 2\pi \int_{u=1}^{u=e} \sec^3\theta\,d\theta$$

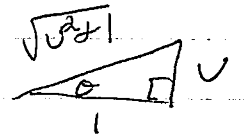
let $u_2 = \sec\theta \qquad dv_2 = \sec^2\theta\,d\theta$

$du_2 = \sec\theta\tan\theta\,d\theta \qquad v_2 = \tan\theta$

$$I = 2\pi\left(\sec\theta\tan\theta\Big|_{u=1}^{u=e} - \int_{u=1}^{u=e} \sec\theta\tan^2\theta\,d\theta\right)$$

$$= \quad '' \qquad '' \qquad - \int_{u=1}^{u=e} (\sec^3\theta - \sec\theta\,d\theta))$$

$$= 2\pi\left(\sec\theta\tan\theta\Big|_{u=1}^{u=e} - I/2\pi + \ln|\sec\theta + \tan\theta|\,\Big|_{u=1}^{u=e}\right)$$



$$I = \pi\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\,\Big|_{u=1}^{u=e}\right)$$

$$= \pi\left(u\sqrt{1+u^2} + \ln|\sqrt{1+u^2} + u|\right)\Big|_1^e$$

$$= \pi\left(e\sqrt{1+e^2} + \ln|\sqrt{1+e^2} + e| - \sqrt{2} - \ln|\sqrt{2} + 1|\right)$$

Hydrostatic Pressure and Force.

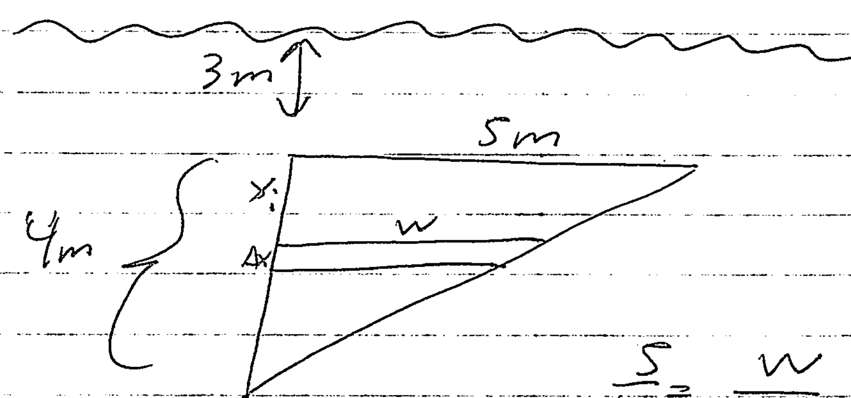
The pressure P on a small portion of a plane is defined to be the force per unit area.

$P = \rho g d$. ρ = density g = gravity acceleration
d = depth.

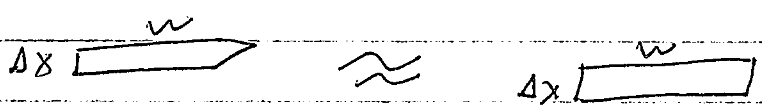
ulless stated otherwise $\rho = 1000 kg/m^3 = 1 g/cm^3$ $g = 9.8 m/s^2$

Hydrostatic force is $F = PA$ A = area of the surface.

Example



$$\frac{5}{4} = \frac{w}{4 - x_i}$$



Hydrostatic Force is $\approx \rho g (3 + x_i)(\frac{5}{4}(4 - x_i)\Delta x)$

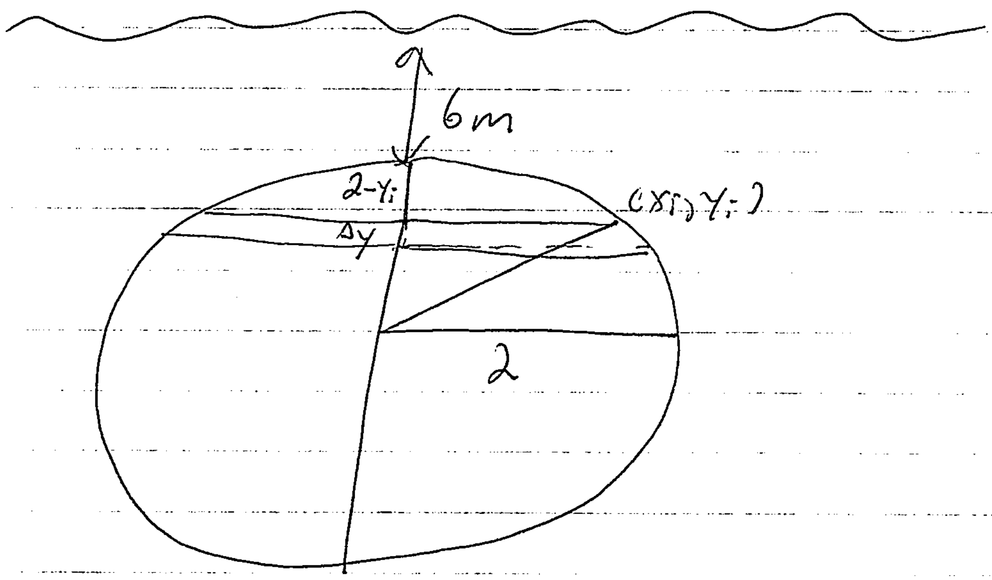
Total " $\approx \sum_{i=1}$ " " "

let $n \to \infty$ then

$$\text{Force} = \frac{5}{4}\int_0^4 \rho g (3 + x)(4 - x)\, dx$$

$$= 1000 \cdot 9.8 \cdot \left(\frac{130}{3}\right) \approx 425 kN$$

Find the hydrostatic force on a circular plate submerged 6m that has a radius of 2m.



Strip Area $\approx 2 \cdot (4-y_i^2)^{1/2} \Delta y$

depth $= 6+(2-y_i) = 8-y_i$

$\int_{-2}^{2} \rho g (8-y) 2 (4-y^2)^{1/2} dy \approx 985 kN$