$$\sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n\sqrt{n+1}}$$

$$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{2^{n+1}\sqrt{n+2}} \cdot \frac{2^n\sqrt{n+1}}{(x+1)^n} \right| = \lim_{n\to\infty} \left| \frac{x+1}{2} \cdot \frac{\sqrt{n+1}}{\sqrt{n+2}} \right|$$

$$= \frac{|x+1|}{2} < 1 \Rightarrow |x+1| < 2$$
$$\Rightarrow -2 < x+1 < 2$$
$$\Rightarrow -3 < x < 1$$

let $x = -3$

$$\sum_{n=0}^{\infty} \frac{(-3+1)^n}{2^n\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{(-2)^n}{2^n\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

converges by alternating series test

let $x = 1$

$$\sum_{n=0}^{\infty} \frac{(1+1)^n}{2^n\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}} > 1 + \sum_{n=1}^{\infty} \frac{1}{2n} \quad \text{or} \quad 1 + \sum_{n=1}^{\infty} \frac{1}{\sqrt{2n}}$$

~~or~~ both of which diverge p-test $p \leq 1$.

$\therefore$ by comparison test $\sum_{n=0}^{\infty} \frac{(1+1)^n}{2^n\sqrt{n+1}}$ diverges

$\therefore$ IC $= [-3, 1)$

$$\sum_{n=0}^{\infty} \frac{n!\, x^n}{1\cdot 3\cdot 5\cdot \ldots \cdot (2n+1)}$$

Power Series of arctan(x)

Ex Express $\frac{1}{1-2x+x^2}$ as a power series

$$\frac{1}{1-2x+x^2} = \frac{1}{(1-x)^2} = +1 \cdot \left(\frac{1}{1-x}\right)' = +1\left(\sum_{n=0}^{\infty} x^n\right)'$$

$$= +1\,(1+x+x^2+\dots)' = +1\,(1+2x+3x^2+4x^3+\dots)$$

$$= +\sum_{n=0}^{\infty}(n+1)x^n \qquad R=1 \qquad IC=(-1,1)$$

Express $\ln(1+x)$ as a power series

$$\frac{d\ln(1+x)}{dx} = \frac{1}{1+x} = \frac{1}{1-(-x)} = 1-x+x^2-x^3+\dots$$

$$\ln(1+x) = C + \int(1-x+x^2-x^3+\dots)$$
$$= C + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

let $x=0$ then

$$\ln(1) = C = 0$$

$$\therefore \ln(1+x) = \sum_{n=0}^{\infty} \frac{x^{n+1}(-1)^n}{n+1}$$

$$R=1 \qquad IC=(-1,1)$$