A general order ODE has the form

$y' = F(x,y)$ here y is a function of x
and F is a function of 2 variable

Ex $y' = x^2 + xy + y^2$ (not seperable)

An equation $y' = F(x,y)$ is <u>seperable</u> if

$F(x,y)$ can be decomposed into $g(x) \cdot f(y)$.

A <u>seperable</u> equation can be solved as follows

$$\frac{dy}{dx} = g(x) f(y) \Rightarrow \frac{dy}{f(y)} = g(x)\,dx$$
$$\Rightarrow \int \frac{1}{f(y)}\,dy = \int g(x)\,dx$$

Example $\frac{dy}{dx} = \frac{x^2}{y^4}$ with $y(0)=2$

Example $\frac{dy}{dx} = \frac{e^x}{\cos(y) + \sin(y) + 2y}$ note unable to solve explicity for y

$\int \cos(y) + \sin(y) + 2y \, dy = \int e^x \, dx$

$\sin(y) - \cos(y) + y^2 = e^x + C$

$\frac{dy}{dx} = xy$

$\int \frac{1}{y} dy = \int x \, dx$ assume $y \neq 0$

$\ln|y| = \frac{x^2}{2} + C$

$|y| = e^{x^2/2} \cdot e^C$

$y = \pm e^C e^{x^2/2} = \pm C_1 e^{x^2/2}$ or $y = 0$

Orthogonal Trajectory

$$\frac{dy}{dx} = F(x, y)$$

slope of the Orthogonal Trajectories

$$\frac{dy}{dx} = \frac{-1}{F(x, y)}$$

For $x = ky^2$

$$\frac{dx}{dx} = k\,2y\,\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2ky} = \frac{1}{2\left(\frac{x}{y^2}\right)y} = \frac{y}{2x}$$

so for the orthogonal Trajectory

$$\frac{dy}{dx} = \frac{-1}{\left(\frac{y}{2x}\right)} = \frac{-2x}{y}$$

$$\Rightarrow 2x\,dx + y\,dy = 0 \Rightarrow x^2 + \frac{y^2}{2} = C$$

a family of elipses.