Taylor and Maclaurin Series

Let us assume that a smooth function f has a power series representation of

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots \qquad |x-a| < R$$

then

$$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \ldots$$
$$f''(x) = 2c_2 + 2\cdot 3\, c_3(x-a) + 3\cdot 4\, c_4(x-a)^2 + \ldots$$
$$f'''(x) = 2\cdot 3\, c_3 + 2\cdot 3\cdot 4\, c_4(x-a) + \ldots$$

$$f(a) = c_0 \qquad f'(a) = c_1 \qquad f''(a) = 2c_2 \qquad f'''(a) = 6c_3$$

Theorem.

If f has a power series ~~pr~~ representation at a

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \qquad |x-a| < R$$

then the coefficients are given by

$$c_n = \frac{f^{(n)}(a)}{n!}$$

$$\therefore\ f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$$

This is called the Taylor series of the function f at a. If $a=0$, then this is a Maclaurin Series.