Average Value

Find the average grade: 80 84 92 86 94 .

Solution: $Average = \dfrac{80+84+92+86+94}{5} = \dfrac{436}{5} = 85.2$

(Sketch)

Average is the sum/(number of terms)

What if there are n number of terms defined by a function $f(x)$ on an interval [a,b]? Then the

average value would be: $\dfrac{\sum\limits_{i=1}^{n} f(x_i)}{n}$.

(Sketch)

If the function f is continuous on [a,b] we define the average value of f to be $\dfrac{\int_a^b f(x)\,dx}{b-a}$

Geometrically, (if the function is positive) it is the area under the curve divided by the length of the curve.

(Sketch)

Example 1: Find the average value of $f(x) = \sin x$ on $[0, \pi]$

Solution:

$$\text{Average value} = \frac{1}{\pi - 0}\int_0^\pi \sin x\,dx = \frac{-\cos x\ \big|_0^\pi}{\pi} = \frac{-(-1)-(-1)}{\pi} = \frac{2}{\pi} \quad \text{[Sketch]}$$

Example 2: Find the average value of $f(x) = 3x^2$ on $[1,5]$.

Solution:

$$\text{Average value} = \frac{1}{5-1}\int_1^5 3x^2\,dx = \frac{x^3\big|_1^5}{4} = \frac{125-1}{4} = \frac{124}{4} = 31 \quad \text{[Sketch]}$$

Example 3: Find the value of b if the average value of $f(x) = x^2 + b$ on [0,2] is 8.