Newton's Law of Cooling

T_s = temperature of surroundings

$T(t)$ = temperature at time t.

$$\frac{dT}{dt} = k(T - T_s) \quad \text{where } k<0.$$

Method 1. $\displaystyle\int \frac{dT}{T-T_s} = \int k\,dt$

Method 2. let $v = T - T_s$ then $\frac{dv}{dt} = \frac{dT}{dt}$

$\frac{dv}{dt} = k(T - T_s) = kv \quad \therefore v(t) = v(0)\, e^{kt}$

$v(0) = T_0 - T_s$.

Example Temperature in Fridge is $7°C$

Dinner at room temperature is $22°C$.

After 30 min in the fridge, dinner

is now $16°C$. Solve for k.

A <u>First Order linear ODE</u> (FOODE)

is an ODE of the following form:

$\frac{dy}{dx} + P(x)y = Q(x)$, where $P(x), Q(x)$ are given functions of x,

$\frac{dy}{dx} = Q(x) - P(x)y = F(x,y)$ This is <u>not seperable</u>

unless $P(x) = a\,Q(x)$ where a is a constant.

To solve this we use an <u>integrating factor</u>

$G(x) = e^{\int P(x)dx}$ then $G'(x) = P(x)e^{\int P(x)dx} = P(x)G(x)$

$$\frac{d}{dx}(y\,G(x)) = \frac{dy}{dx}G(x) + y\,P(x)\,G(x).$$
$$= G(x)\left(\frac{dy}{dx} + y\,P(x)\right)$$
$$= G(x)\,Q(x)$$

$\therefore$ $y\,G(x) = \int G(x)\,Q(x)\,dx + C$

$$y = \frac{1}{G(x)}\left(\int G(x)\,Q(x)\,dx + C\right)$$

Example $y' + \frac{1}{x}y = 7$ $\quad P(x) = 1/x \quad Q(x) = 7$

$$G(x) = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = |x|$$

These are usually populations, so it is assumed $x \geq 0$,

$G(x) = x$

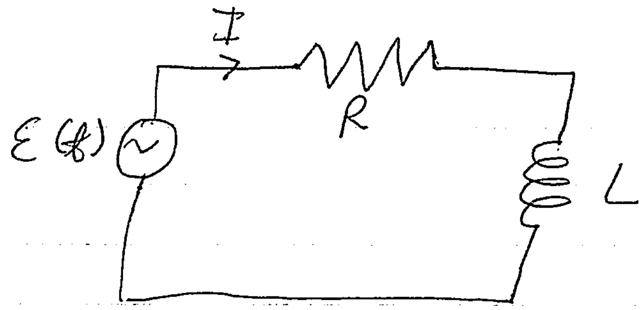
$$y = \frac{1}{x}\left(\int x \cdot 7 \cdot dx + C\right) = \frac{1}{x}\left(\frac{7x^2}{2} + C\right)$$

$$= \frac{7x}{2} + \frac{C}{x}$$

Ex $y' + 3x^2 y = 7x^3$ $\quad$ (seperable). $P(x) = 3x^2 \quad Q(x) = 7x^3$

$$G(x) = e^{\int 3x^2 dx} = e^{x^3}$$

$y = ?$



R = resistance in ohms
L = inductance in henrys
$\mathcal{E}(t)$ = electromotive force in volts
$I(t)$ = current in amps

Kirchoff's Voltage Law: The directed sum of electrical potential differences around any closed circuit is zero.

$$IR + L\frac{dI}{dt} = \mathcal{E}(t)$$

$$\frac{dI}{dt} + \left(\frac{R}{L}\right) I = \frac{\mathcal{E}(t)}{L}$$

If R and L are not time dependent (independent of t), then the integrating factor is:

$$e^{\int R/L \, dt} = e^{(Rt)/L}$$

$$\therefore I = \frac{1}{e^{(Rt)/L}}\left(\int e^{\frac{R}{L}t} \frac{\mathcal{E}(t)}{L} dt + C\right)$$

$$= \frac{e^{-(Rt)/L}}{L} \int e^{\frac{Rt}{L}} \mathcal{E}(t)\, dt + Ce^{-\frac{(Rt)}{L}}$$

$Ce^{-\frac{Rt}{L}}$ is called a transient as it goes to 0 as $t \to \infty$.