Function of several variables

Let n be a positive integer. Then a <u>function of n-variables</u> is a rule that assigns a real number to each n-tuple in a given set $D \subset \mathbb{R}^n$. Where D is called the <u>domain</u> of f

$(x_1, x_2, \ldots, x_n) \rightarrow f(x_1, x_2, \ldots, x_n)$
input output

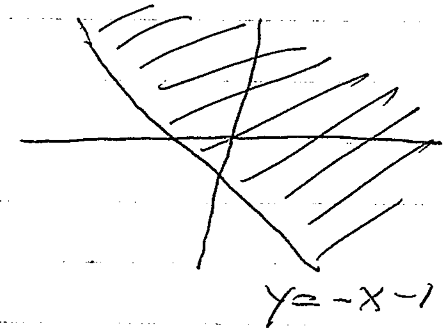
The "natural domain" is the largest domain that makes sense ~~for~~ for the given function.

Ex. $f(x,y) = \ln(1+x+y)$

$$D = \{(x,y) \mid 1+x+y > 0\}$$

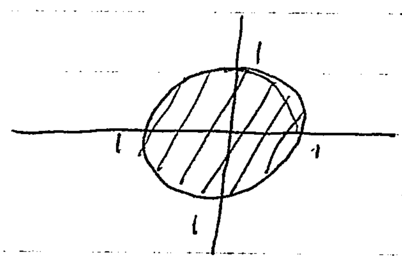
D ~~ann~~ is the points (x,y) such that

$1+x+y > 0$. $y > -1-x$



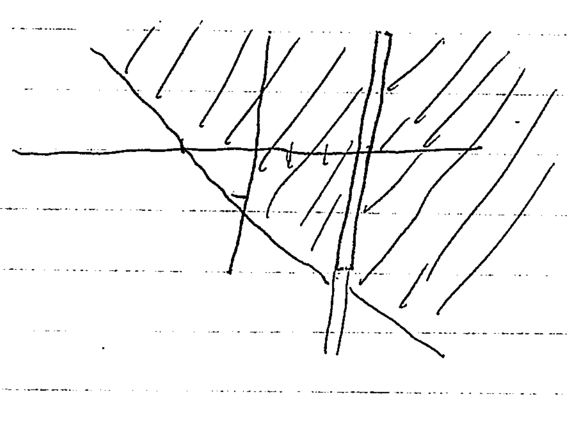
$$f(x,y) = (1-(x^2+y^2))^{1/2}$$

$$D = \{(x,y) \mid x^2+y^2 \le 1\}$$



$$f(x,y) = \frac{\ln(1+x+y)}{x-3}$$

$$D = \{(x,y) \mid 1+x+y > 0 \text{ and } x \neq 3$$



The range of a function $f(x_1, x_2, \ldots, x_n)$ with a domain D is the set of possible values of $f(x_1, \ldots, x_n)$ as $(x_1, \ldots, x_n)$ ranges over D.

Ex:

1) $\cos(x+y) = f(x,y)$

   Range $= [-1, 1] = \{z \mid -1 \le z \le 1\}$

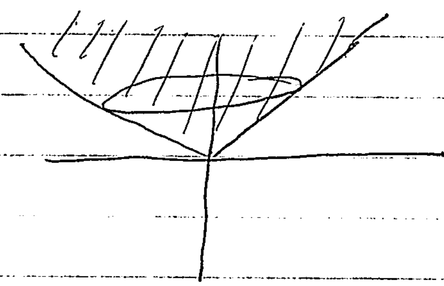
2) range of $f(x,y) = e^{x^2 - y^2}$;

   Range is $(0, \infty) = \{z \mid z > 0\}$

3) $f(x,y) = ax + by$ where a, b are both non-zero constants.

$f(x,y) = (x^2+y^2)^{1/2}$

$D = \mathbb{R}$ Range =



$f(x,y) = (1-(x^2+y^2))^{1/2}$

$D = \{(x,y) \mid x^2+y^2 \leq 1\}$

Range = The northern hemisphere of a sphere of radius 1 centered at (0,0)