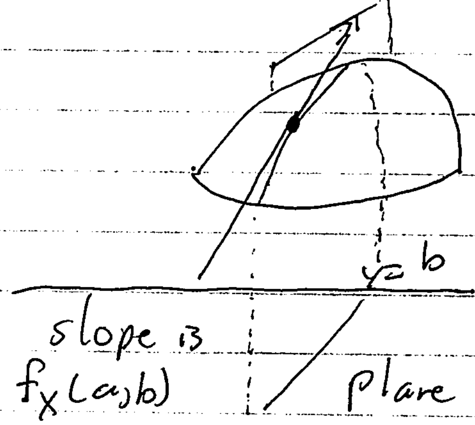
# Geometric Interpretation




$g(x) = f(x, b)$

$f_x(a, b) = g'(a) =$
slope of tangent to $g(x) = f(x, b)$
at $x = a$.

plane $y = b$ parallel to the $xz$-coordinate plane.

for $f_y(a, b) = h'(b) =$ slope of tangent to
$h(y) = f(a, b)$ at $y = b$

take slice at $x = a$, parallel to the $yz$-coordinate plane.

# Implicit differentiation

z is a function of x, y

$x^3 + y^3 + z^3 + xyz + xy = 1$ find partial wrt x.

$3x^2 + 0 + 3z^2 \frac{dz}{dx} + yz + xy\frac{dz}{dx} + y = 0$

$\frac{dz}{dx}(3z^2 + xy) = -3x^2 - yz - y$

$\frac{dz}{dx} = \frac{-3x^2 - yz - y}{3z^2 + xy}$

Functions with more than two variables.

$w = f(x_1, \ldots, x_n)$

$$\frac{\partial w}{\partial x_i} = \frac{\partial f}{\partial x_i}$$

Example $x^2 e^{y^2+z}$

$$\frac{\partial f}{\partial x} = 2x e^{y^2+z}$$

$$\frac{\partial f}{\partial y} = x^2 2y e^{y^2+z}$$

$$\frac{\partial f}{\partial z} = x^2 e^{y^2+z}$$

Higher derivatives

$$(f_x)_x = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$(f_x)_y = f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\,\partial x}$$ right to left

left to right

$$(f_y)_x = f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\,\partial y}$$

$$(f_y)_y = f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

ex. $f(x,y) = x\sin(y) + ye^x$

$f_x = \sin(y) + ye^x$ $\qquad$ $f_y = x\cos(y) + e^x$

$f_{xx} = ye^x$ $\qquad$ $f_{yy} = -x\sin(y)$

$f_{xy} = \cos(y) + e^x$ $\qquad$ $f_{yx} = \cos(y) + e^x$

$f_{xy} = f_{yx}$

Not always the case.

# Clairaut's Theorem

Let $U = \{(x,y) \mid (x-a)^2 + (y-b)^2 < r^2\}$, which is the open disk of radius $r$ centered at $(a,b)$. If $f(x,y)$ is a real valued function defined on $U$ such that $f_{xy}$ and $f_{yx}$ both exist and are continuous on $U$ then $f_{xy} = f_{yx}$ on $U$.