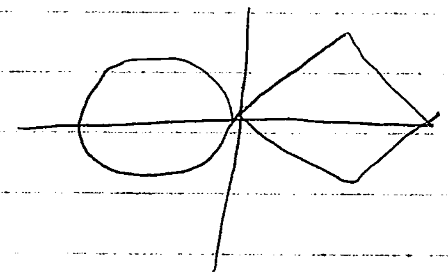
Symmetries of the polar equation

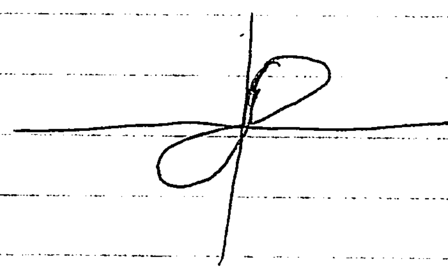
1) If $f(-\theta) = f(\theta)$ symmetric about the polar axis

ex:



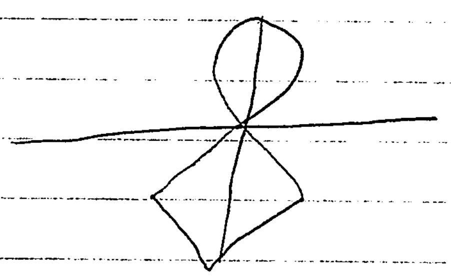
2) If $f(\theta) = f(\theta + \pi)$
unchanged if you rotate $180°$ or π.

ex:



3. $f(\theta) = f(\pi - \theta)$ symmetric about the line $\theta = \pi/2$ (or y axis)

Ex



Tangents to Polar curves

$x = r\cos\theta = f(\theta)\cos\theta$

$y = r\sin\theta = g(\theta)\sin\theta$

∴ treat it exactly like parametric

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

cardiod

$r = 1 + \sin\theta$

$x = \cos\theta + \sin\theta\cos\theta$

$y = \sin\theta + \sin^2\theta$

or use formula above

$$\frac{dy}{dx} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta + \cos^2\theta - \sin^2\theta}$$

$$= \frac{\cos\theta(1 + 2\sin\theta)}{-\sin\theta + (1 - \sin^2\theta) - \sin^2\theta}$$

$$= \frac{\cos\theta(1 + 2\sin\theta)}{1 - \sin\theta - 2\sin^2\theta} = \frac{\cos\theta(1 + 2\sin\theta)}{(1 + \sin\theta)(1 - 2\sin\theta)}$$

vertical tangent again when $\frac{dy}{dx}$ divides by 0

so $\sin\theta = +1$ or $\sin\theta = 1/2$

$\theta = -3\pi/2$ $\quad\quad$ $\theta = \pi/6, 5\pi/6$

horizontal tangent when $\frac{dy}{dx} = 0$

$\cos\theta = 0$ $\quad\quad$ $\sin\theta = -1/2$

$\theta = \pi/2, 3\pi/2$ $\quad\quad$ $\theta = 7\pi/6, 11\pi/6$