Solutions to Parametric Sketches

Solution:

1) Intercepts

$y = 0$	$x = 0$
$0 = 3t^2 - 9$	$0 = t^3 - 3t$
$0 = 3\left(t^2 - 3\right)$	$0 = t\left(t - \sqrt{3}\right)\left(t + \sqrt{3}\right)$ $\therefore$ the curve crosses itself
$t = \pm\sqrt{3}$	$t = 0, \sqrt{3}, -\sqrt{3}$ at $(0,0)$ when $t = \pm\sqrt{3}$
$\therefore x-$intercepts are: 0	$y-$intercepts are: $0, -9$

Beginning and Ending Positions

$$\lim_{t\to\pm\infty} x = \lim_{t\to\pm\infty}\left(t^3 - 3t\right) = \lim_{t\to\pm\infty}\left[t\left(t^2 - 3\right)\right] = \pm\infty$$

$$\lim_{t\to\pm\infty} y = \lim_{t\to\pm\infty}\left(3t^2 - 9\right) = \infty$$

Coordinates of points of horizontal and vertical tangent lines.

$\frac{dx}{dt} = 3t^2 - 3$	$\frac{dy}{dt} = 6t$
$0 = 3(t-1)(t+1)$	$0 = 6t$
$t = 1, -1$ and $\frac{dy}{dt} \neq 0$	$t = 0$ and $\frac{dx}{dt} \neq 0$
$\therefore$ The curve has vertical tangents at $(-2,-6)$ and $(2,-6)$	$\therefore$ The curve has horizontal tangents at $(0,-9)$

The Chart

We will use the critical values of t to determine the directions of the curve

t	$t<-1$	$-1<t<0$	$0<t<1$	$t>1$
$\frac{dx}{dt}$	+	–	−	+
$\frac{dy}{dt}$	−	−	+	+
x	→	←	←	→
y	↓	↓	↑	↑
curve	↘	↙	↖	↗

Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{6t}{3t^2-3}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d\left(\frac{6t}{3t^2-3}\right)}{dt}}{\frac{dx}{dt}}=\frac{\frac{6\left(3t^2-3\right)-6t\left(6t\right)}{\left(3t^2-3\right)^2}}{3t^2-3}$$

$$=\frac{18t^3-36t^2-18t^3+18t^2+18t-18}{\left(3t^2-3\right)^3}$$

$$=\frac{-18t^2-18}{\left(3t^2-3\right)^3}=\frac{-18\left(t^2+1\right)}{27\left(t-1\right)^3\left(t+1\right)^3}$$

Therefore, the numbers which appear on the number line are $t=-1$ and 1.

t		-1		1	
$\frac{d^2y}{dx^2}$	$-$		$+$		$-$
curve	$\cap$		$\cup$		$\cap$

2) Intercepts

$x=0$	$y=0$	
$0=t^2-4$	$0=t^3-4t$	
$0=\left(t-2\right)\left(t+2\right)$	$0=t\left(t-2\right)\left(t+2\right)$	$\therefore$ the curve crosses itself
$t=\pm 2$	$t=0,\pm 2$	at $\left(0,0\right)$ when $t=\pm 2$
$\therefore$ $y-$intercepts are: 0	$x-$intercepts are: $0,-4$	

Beginning and Ending Positions

$$\lim_{t\to\pm\infty} x=\lim_{t\to\pm\infty}\left(t^2-4\right)=\infty$$

$$\lim_{t\to\pm\infty} y=\lim_{t\to\pm\infty}\left(t^3-4t\right)=\lim_{t\to\pm\infty} t\left(t^2-4\right)=\pm\infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dy}{dt} = 3t^2 - 4 \qquad\qquad \frac{dx}{dt} = 2t$$

$$0 = \left(\sqrt{3}t - 2\right)\left(\sqrt{3}t + 2\right) \qquad\qquad 0 = 2t$$

$$t = \pm\frac{2}{\sqrt{3}} \text{ and } \frac{dx}{dt} \neq 0 \qquad\qquad t = 0 \text{ and } \frac{dy}{dt} \neq 0$$

$\therefore$ The curve has horizontal tangents at $\left(-\frac{8}{3}, -\frac{16}{3\sqrt{3}}\right)$ and $\left(-\frac{8}{3}, \frac{16}{3\sqrt{3}}\right)$

$\therefore$ The curve has vertical tangent at $(-4, 0)$

The Chart

We will use the critical values of t to determine the directions of the curve

t		$-\frac{2}{\sqrt{3}}$		0		$\frac{2}{\sqrt{3}}$	
$\frac{dx}{dt}$	$-$		$-$		$+$		$+$
$\frac{dy}{dt}$	$+$		$-$		$-$		$+$
x	$\leftarrow$		$\leftarrow$		$\rightarrow$		$\rightarrow$
y	$\uparrow$		$\downarrow$		$\downarrow$		$\uparrow$
curve	$\nwarrow$		$\swarrow$		$\searrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 4}{2t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{3t^2-4}{2t}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{6t(2t) - 2\left(3t^2 - 4\right)}{(2t)^2}}{2t} = \frac{3t^2 + 4}{4t^3}$$

Therefore, the number which appear on the number line is $t = 0$.

t		0	
$\frac{d^2y}{dx^2}$	$-$		$+$
curve	$\cap$		$\cup$

3) <u>Intercepts</u>

$x=0$

$0=t^2-t$

$0=t(t-1)$

$t=0,1$

$\therefore$ $y-$intercepts are: $0,-2$

$y=0$

$0=t^3-3t$

$0=t\left(t-\sqrt{3}\right)\left(t+\sqrt{3}\right)$

$t=0,\pm\sqrt{3}$

$x-$intercepts are: $0, 3-\sqrt{3}, 3+\sqrt{3}$

<u>Beginning and Ending Positions</u>

$$\lim_{t\to\pm\infty} x = \lim_{t\to\pm\infty}\left(t^2-t\right) = \lim_{t\to\pm\infty} t(t-1) = \infty$$

$$\lim_{t\to\pm\infty} y = \lim_{t\to\pm\infty}\left(t^3-3t\right) = \lim_{t\to\pm\infty} t\left(t^2-3\right) = \pm\infty$$

<u>Coordinates of points of horizontal and vertical tangent lines.</u>

$\frac{dy}{dt}=3t^2-3$

$0=3(t-1)(t+1)$

$t=\pm1$ and $\frac{dx}{dt}\neq 0$

$\therefore$ The curve has horizontal tangents at $(0,-2)$ and $(2,2)$

$\frac{dx}{dt}=2t-1$

$0=2t-1$

$t=\frac{1}{2}$ and $\frac{dy}{dt}\neq 0$

$\therefore$ The curve has vertical tangent at $\left(-\frac{1}{4},-\frac{11}{8}\right)$

<u>The Chart</u>

We will use the critical values of *t* to determine the directions of the curve

t		-1		$\frac{1}{2}$		1	
$\frac{dx}{dt}$	$-$		$-$		$+$		$+$
$\frac{dy}{dt}$	$+$		$-$		$-$		$+$
x	$\leftarrow$		$\leftarrow$		$\rightarrow$		$\rightarrow$
y	$\uparrow$		$\downarrow$		$\downarrow$		$\uparrow$
curve	$\nwarrow$		$\swarrow$		$\searrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

<u>Check the concavity</u>

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3t^2-3}{2t-1}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d\left(\frac{3t^2-3}{2t-1}\right)}{dt}}{\frac{dx}{dt}}=\frac{\frac{6t(2t-1)-2(3t^2-3)}{(2t-1)^2}}{2t-1}=\frac{6(t^2-t+1)}{(2t-1)^3}$$

Therefore, the number which appear on the number line is $t=\frac{1}{2}$.

t		$\frac{1}{2}$	
$\frac{d^2y}{dx^2}$	$-$		$+$
$curve$	$\cap$		$\cup$

4) <u>Intercepts</u>

$y=0$	$x=0$
$0=t^2+t$	$0=t^2+2t$
$0=t(t+1)$	$0=t(t+2)$
$t=0,-1$	$t=0,-2$
$\therefore x-$intercepts are: $0,-1$	$y-$intercepts are: $0,2$

<u>Beginning and Ending Positions</u>

$$\lim_{t\to\pm\infty} x=\lim_{t\to\pm\infty}\left(t^2+2t\right)=\lim_{t\to\pm\infty}\left[t(t+2)\right]=\infty$$

$$\lim_{t\to\pm\infty} y=\lim_{t\to\pm\infty}\left(t^2+t\right)=\lim_{t\to\pm\infty} t(t+1)=\infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 2t + 2$$

$$0 = 2(t+1)$$

$$t = -1 \text{ and } \frac{dy}{dt} \neq 0$$

$\therefore$ The curve has vertical tangent at $(-1, 0)$

$$\frac{dy}{dt} = 2t + 1$$

$$0 = 2t + 1$$

$$t = -\frac{1}{2} \text{ and } \frac{dx}{dt} \neq 0$$

$\therefore$ The curve has horizontal tangent at $\left(-\frac{3}{4}, -\frac{1}{2}\right)$

The Chart

We will use the critical values of *t* to determine the directions of the curve

t		-1		$\frac{-1}{2}$	
$\frac{dx}{dt}$	$-$		$+$		$+$
$\frac{dy}{dt}$	$-$		$-$		$+$
x	$\leftarrow$		$\rightarrow$		$\rightarrow$
y	$\downarrow$		$\downarrow$		$\uparrow$
curve	$\swarrow$		$\searrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t+2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{2t+1}{2t+2}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{2(2t+2)-2(2t+1)}{(2t+2)^2}}{2t+2}$$

$$= \frac{2}{(2t+2)^3}$$

Therefore, the number which appears on the number line is $t = -1$.

t		-1	
$\frac{d^2y}{dx^2}$	$-$		$+$
curve	$\cap$		$\cup$

5) <u>Intercepts</u>

$y = 0$

$0 = t^2 - 2t$

$0 = t(t-2)$

$t = 0, 2$

$\therefore x-$intercepts are: $0, 4$

$x = 0$

$0 = t^2$

$0 = 0$

$y-$intercepts are: 0

<u>Beginning and Ending Positions</u>

$$\lim_{t\to\pm\infty} x = \lim_{t\to\pm\infty} t^2 = \infty$$

$$\lim_{t\to\pm\infty} y = \lim_{t\to\pm\infty} \left(t^2 - 2t\right) = \lim_{x\to\pm\infty} t(t-2) = \infty$$

<u>Coordinates of points of horizontal and vertical tangent lines.</u>

$\frac{dx}{dt} = 2t$

$0 = 2t$

$t = 0$ and $\frac{dy}{dt} \neq 0$

$\therefore$ The curve has vertical tangent at $(0,0)$

$\frac{dy}{dt} = 2t - 2$

$0 = 2t - 2$

$t = 1$ and $\frac{dx}{dt} \neq 0$

$\therefore$ The curve has horizontal tangent at $(1,-1)$

<u>The Chart</u>

We will use the critical values of t to determine the directions of the curve

t		0		1	
$\frac{dy}{dt}$	$-$		$-$		$+$
$\frac{dx}{dt}$	$-$		$+$		$+$
x	$\leftarrow$		$\rightarrow$		$\rightarrow$
y	$\downarrow$		$\downarrow$		$\uparrow$
curve	$\swarrow$		$\searrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2t-2}{2t}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d\left(\frac{2t-2}{2t}\right)}{dt}}{\frac{dx}{dt}}=\frac{\frac{2(2t)-2(2t-2)}{(2t)^2}}{2t}$$

$$=\frac{4}{(2t)^3}$$

Therefore, the number which appears on the number line is $t = 0$.

t		0	
$\frac{d^2y}{dx^2}$	$-$		$+$
curve	$\cap$		$\cup$

6) Intercepts

$y=0$	$x=0$
$0=t^2+t-2$	$0=t^2+2t-1$
$0=(t+2)(t-1)$	$t=\frac{-2\pm\sqrt{2^2+4}}{2}$
$t=1,-2$	$t=-1\pm\sqrt{2}$
$\therefore$ $x-$intercepts are: $-1,2$	$y-$intercepts are: $\pm\sqrt{2}$

Beginning and Ending Positions

$$\lim_{t\to\pm\infty}x=\lim_{t\to\pm\infty}\left(t^2+2t-1\right)=\infty$$

$$\lim_{t\to\pm\infty}y=\lim_{t\to\pm\infty}\left(t^2+t-2\right)=\infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 2t + 2$$
$$0 = 2(t+1)$$
$$t = -1 \text{ and } \frac{dy}{dt} \neq 0$$
$\therefore$ The curve has vertical tangent at $(-2,-2)$

$$\frac{dy}{dt} = 2t + 1$$
$$0 = 2t + 1$$
$$t = -\frac{1}{2} \text{ and } \frac{dx}{dt} \neq 0$$
$\therefore$ The curve has horizontal tangent at $\left(-\frac{7}{4},-\frac{9}{4}\right)$

The Chart

We will use the critical values of *t* to determine the directions of the curve

t		-1		$\frac{-1}{2}$	
$\frac{dx}{dt}$	$-$		$+$		$+$
$\frac{dy}{dt}$	$-$		$-$		$+$
x	←		→		→
y	↓		↓		↑
curve	↙		↘		↗

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t+2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{2t+1}{2t+2}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{2(2t+2)-2(2t+1)}{(2t+2)^2}}{2t+2}$$

$$= \frac{2}{(2t+2)^3}$$

Therefore, the number which appears on the number line is $t = -1$.

t		-1	
$\dfrac{d^2y}{dx^2}$	$-$		$+$
curve	$\cap$		$\cup$

7) <u>Intercepts</u>

$y = 0$

$0 = t^3 - 3t^2$

$0 = t^2(t-3)$

$t = 0, 3$

$\therefore$ $x-$intercepts are: $0, 18$

$x = 0$

$0 = t^3 - 3t$

$0 = t\left(t-\sqrt{3}\right)\left(t+\sqrt{3}\right)$

$t = 0, \sqrt{3}, -\sqrt{3}$

$y-$intercepts are: $0, \pm 3\sqrt{3} - 9$

<u>Beginning and Ending Positions</u>

$$\lim_{t\to\pm\infty} x = \lim_{t\to\pm\infty}\left(t^3 - 3t\right) = \lim_{t\to\pm\infty}\left[t\left(t^2-3\right)\right] = \pm\infty$$

$$\lim_{t\to\pm\infty} y = \lim_{t\to\pm\infty}\left(t^3 - 3t^2\right) = \lim_{t\to\pm\infty}\left[t^2\left(t-3\right)\right] = \pm\infty$$

<u>Coordinates of points of horizontal and vertical tangent lines.</u>

$\dfrac{dx}{dt} = 3t^2 - 3$

$0 = 3(t-1)(t+1)$

$t = 1, -1$ and $\dfrac{dy}{dt} \neq 0$

$\therefore$ The curve has vertical tangents at $(-2,-2)$ and $(2,-4)$

$\dfrac{dy}{dt} = 3t^2 - 6t$

$0 = 3t(t-2)$

$t = 0, 2$ and $\dfrac{dx}{dt} \neq 0$

$\therefore$ The curve has horizontal tangents at $(0,0)$ and $(2,-4)$

Therefore, the curve crosses itself at $(2,-4)$ when $t = -1$ and $t = 2$.

<u>The Chart</u>

We will use the critical values of t to determine the directions of the curve

t		-1		0		1		2	
$\dfrac{dx}{dt}$	$+$		$-$		$-$		$+$		$+$
$\dfrac{dy}{dt}$	$+$		$+$		$-$		$-$		$+$
x	$\rightarrow$		$\leftarrow$		$\leftarrow$		$\rightarrow$		$\rightarrow$
y	$\uparrow$		$\uparrow$		$\downarrow$		$\downarrow$		$\uparrow$
curve	$\nearrow$		$\nwarrow$		$\swarrow$		$\searrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3t^2-6t}{3t^2-3}=\frac{t^2-2t}{t^2-1}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d\left(\frac{t^2-2t}{t^2-1}\right)}{dt}}{\frac{dx}{dt}}=\frac{\frac{(2t-2)(t^2-1)-2t(t^2-2t)}{(t^2-1)^2}}{3t^2-3}$$

$$=\frac{2(t^2-t+1)}{3(t^2-1)^3}$$

$$=\frac{2(t^2-t+1)}{3(t-1)^3(t+1)^3}$$

Therefore, the numbers which appear on the number line are $t=-1$ and 1.

t		-1		1	
$\frac{d^2y}{dx^2}$	$+$		$-$		$+$
curve	$\cup$		$\cap$		$\cup$

8) Intercepts

$y=0$	$x=0$
$0=t^2-2t$	$0=t^3$
$0=t(t-2)$	$t=0$
$t=0,2$	$y-$intercepts are: 0
$\therefore x-$intercepts are: $0,8$	

Beginning and Ending Positions

$$\lim_{t\to\pm\infty} x=\lim_{t\to\pm\infty} t^3=\pm\infty$$

$$\lim_{t\to\pm\infty} y=\lim_{t\to\pm\infty}(t^2-2t)=\lim_{t\to\pm\infty}\left[t(t-2)\right]=\infty$$

Coordinates of points of horizontal and vertical tangent lines.

$\frac{dx}{dt} = 3t^2$

$0 = 3t^2$

$t = 0$ and $\frac{dy}{dt} \neq 0$

$\therefore$ The curve has vertical tangents at $(0,0)$

$\frac{dy}{dt} = 2t - 2$

$0 = 2(t-1)$

$t = 1$ and $\frac{dx}{dt} \neq 0$

$\therefore$ The curve has horizontal tangents at $(1,-1)$

The Chart

We will use the critical values of *t* to determine the directions of the curve

t		0		1	
$\frac{dy}{dt}$	$-$		$-$		$+$
$\frac{dx}{dt}$	$+$		$+$		$+$
x	$\rightarrow$		$\rightarrow$		$\rightarrow$
y	$\downarrow$		$\downarrow$		$\uparrow$
curve	$\searrow$		$\searrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-2}{3t^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{2t-2}{3t^2}\right)}{dt}}{\frac{dx}{dt}} = \frac{2\left(3t^2\right) - 6t\left(2t-2\right)}{\left(3t^2\right)^3} = \frac{4-2t}{9t^5}$$

Therefore, the number which appear on the number line is *t* = 0.

t		0		2	
$\frac{d^2y}{dx^2}$	$-$		$+$		$-$
curve	$\cap$		$\cup$		$\cap$

9) <u>Intercepts</u>

$y=0$	$x=0$
$0=t^2-2t$	$0=t^3-3t$
$0=t(t-2)$	$0=t\left(t-\sqrt{3}\right)\left(t+\sqrt{3}\right)$
$t=0,2$	$t=0,\sqrt{3},-\sqrt{3}$
$\therefore x-$intercepts are: $0,2$	$y-$intercepts are: $0, 3\pm 2\sqrt{3}$

<u>Beginning and Ending Positions</u>

$$\lim_{t\to\pm\infty} x = \lim_{t\to\pm\infty}\left(t^3-3t\right) = \lim_{t\to\pm\infty}\left[t\left(t^2-3\right)\right] = \pm\infty$$

$$\lim_{t\to\pm\infty} y = \lim_{t\to\pm\infty}\left(t^2-2t\right) = \lim_{t\to\pm\infty} t(t-2) = \infty$$

<u>Coordinates of points of horizontal and vertical tangent lines.</u>

$\frac{dx}{dt} = 3t^2-3$

$0 = 3(t-1)(t+1)$

$t=1,-1$ and $\frac{dy}{dt}\neq 0$ when $t=-1$

But $\frac{dy}{dt}=0$ when $t=1$

$\therefore$ The curve has vertical tangents at $(2,3)$

$\frac{dy}{dt} = 2t-2$

$0 = 2t-2$

$t=1$ and $\frac{dx}{dt}=0$

What happens when t=1?

$$\frac{dy}{dx} = \frac{2t-2}{3t^2-3}\left[\frac{0}{0} \text{ when t=1}\right]$$

$$\underset{=}{[H]}\lim_{t\to 1}\frac{2}{6t} = \frac{1}{3}$$

ie. When $t=1$ the slope of the tangent approaches $\frac{1}{3}$.

<u>The Chart</u>

We will use the critical values of t to determine the directions of the curve

t		-1		1	
$\frac{dx}{dt}$	$+$		$-$		$+$
$\frac{dy}{dt}$	$-$		$-$		$+$
x	$\rightarrow$		$\leftarrow$		$\rightarrow$
y	$\downarrow$		$\downarrow$		$\uparrow$
curve	$\searrow$		$\swarrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2t-2}{3t^2-3}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d\left(\frac{2t-2}{3t^2-3}\right)}{dt}}{\frac{dx}{dt}}=\frac{\frac{2\left(3t^2-3\right)-6t\left(2t-2\right)}{\left(3t^2-3\right)^2}}{3t^2-3}$$

$$=\frac{-6\left(t-1\right)^2}{27\left(t^2-1\right)^3}$$

$$=\frac{-6\left(t-1\right)^2}{27\left(t-1\right)^3\left(t+1\right)^3}$$

Therefore, the numbers which appear on the number line are $t=-1$ and 1.

t		-1		1	
$\frac{d^2y}{dx^2}$	$-$		$+$		$-$
curve	$\cap$		$\cup$		$\cap$

10) Intercepts

$y=0$	$x=0$
$0=4e^{\frac{t}{2}}$	$0=e^t-t$
$\varnothing$	$t=e^t$

$\varnothing$, since the curves $y=x$ and $y=e^x$ do not intersect.

There are no intercepts.

Beginning and Ending Positions

$$\lim_{t\to\infty} x = \lim_{t\to\infty}\left(e^t - t\right) = \lim_{t\to\infty} t\left(\frac{e^t}{t} - 1\right) \qquad \text{But } \lim_{t\to\infty}\left(\frac{e^t}{t}\right) \quad \left[\frac{\infty}{\infty}\right]$$

$$\overset{[H]}{=} \lim_{t\to\infty}\left(\frac{e^t}{t}\right) = \lim_{t\to\infty}\left(\frac{e^t}{1}\right) = \infty$$

$$\therefore \lim_{t\to\infty} x = \lim_{t\to\infty} t\left(\frac{e^t}{t} - 1\right) = \infty\cdot\infty = \infty$$

$$\lim_{t\to-\infty} x = \lim_{t\to-\infty}\left(e^t - t\right) = \lim_{t\to\infty}\left(\frac{1}{e^t} + t\right) = 0 + \infty = \infty$$

$$\lim_{t\to\infty} y = \lim_{t\to\infty} 4e^{\frac{t}{2}} = \infty \quad \text{and} \quad \lim_{t\to-\infty} y = \lim_{t\to-\infty} 4e^{\frac{t}{2}} = 0$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = e^t - 1 \qquad\qquad \frac{dy}{dt} = 4e^{\frac{t}{2}}\left(\frac{1}{2}\right)$$

$$0 = e^t \quad -1 \qquad\qquad 0 = 4e^{\frac{t}{2}}\left(\frac{1}{2}\right)$$

$$e^t = 1 \qquad\qquad \varnothing$$

$t = 0$ and $\frac{dy}{dt} \neq 0$

$\therefore$ The curve has vertical tangents at $(1, 4)$.

The Chart

We will use the critical values of *t* to determine the directions of the curve

t		0	
$\frac{dy}{dt}$	$+$		$+$
$\frac{dx}{dt}$	$-$		$+$
x	$\leftarrow$		$\rightarrow$
y	$\uparrow$		$\uparrow$
curve	$\nwarrow$		$\nearrow$

Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2e^{\frac{t}{2}}}{e^t-1}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d\left(\frac{2e^{\frac{t}{2}}}{e^t-1}\right)}{dt}}{\frac{dx}{dt}}=\frac{\frac{e^{\frac{t}{2}}\left(e^t-1\right)-e^t\cdot 2e^{\frac{t}{2}}}{\left(e^t-1\right)^2}}{e^t-1}=\frac{-e^{\frac{t}{2}}\left(e^t-1-2e^t\right)}{\left(e^t-1\right)^3}$$

$$=\frac{-e^{\frac{t}{2}}\left(1+e^t\right)}{\left(e^t-1\right)^3}$$

$\therefore$ if $e^t-1=0$ then $t=0$

Therefore, the number which appear on the number line is t = 0.

t		0	
$\frac{d^2y}{dx^2}$	+		−
curve	$\cup$		$\cap$