

Solutions to Parametric Sketches

Solution:

1) Intercepts

$$y = 0 \quad x = 0$$

$$0 = 3t^2 - 9 \quad 0 = t^3 - 3t$$

$$0 = 3(t^2 - 3) \quad 0 = t(t - \sqrt{3})(t + \sqrt{3}) \quad \therefore \text{the curve crosses itself}$$

$$t = \pm\sqrt{3} \quad t = 0, \sqrt{3}, -\sqrt{3} \quad \text{at } (0,0) \text{ when } t = \pm\sqrt{3}$$

$$\therefore x\text{-intercepts are: } 0 \quad y\text{-intercepts are: } 0, -9$$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^3 - 3t) = \lim_{t \rightarrow \pm\infty} [t(t^2 - 3)] \xrightarrow{\pm\infty}$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (3t^2 - 9) = \infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 3t^2 - 3 \quad \frac{dy}{dt} = 6t$$

$$0 = 3(t-1)(t+1) \quad 0 = 6t$$

$$t = 1, -1 \text{ and } \frac{dy}{dt} \neq 0 \quad t = 0 \text{ and } \frac{dx}{dt} \neq 0$$

\therefore The curve has vertical tangents at $(-2, -6)$ and $(2, -6)$ \therefore The curve has horizontal tangents at $(0, -9)$

The Chart

We will use the critical values of t to determine the directions of the curve

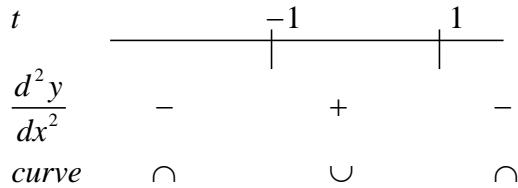
t	-1	0	1	
$\frac{dx}{dt}$	+	-	-	+
$\frac{dy}{dt}$	-	-	+	+
x	\rightarrow	\leftarrow	\leftarrow	\rightarrow
y	\downarrow	\downarrow	\uparrow	\uparrow
<i>curve</i>	\searrow	\swarrow	\nwarrow	\nearrow

Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\begin{aligned}
\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{3t^2 - 3} \\
\frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{6t}{3t^2 - 3}\right)}{\frac{dx}{dt}} = \frac{\frac{6(3t^2 - 3) - 6t(6t)}{(3t^2 - 3)^2}}{\frac{3t^2 - 3}{dt}} \\
&= \frac{18t^3 - 36t^2 - 18t^3 + 18t^2 + 18t - 18}{(3t^2 - 3)^3} \\
&= \frac{-18t^2 - 18}{(3t^2 - 3)^3} = \frac{-18(t^2 + 1)}{27(t-1)^3(t+1)^3}
\end{aligned}$$

Therefore, the numbers which appear on the number line are $t = -1$ and 1 .



2) Intercepts

$$x = 0$$

$$y = 0$$

$$0 = t^2 - 4$$

$$0 = t^3 - 4t$$

$$0 = (t-2)(t+2)$$

$$0 = t(t-2)(t+2) \quad \therefore \text{the curve crosses itself}$$

$$t = \pm 2$$

$$t = 0, \pm 2$$

at $(0,0)$ when $t = \pm 2$

\therefore y -intercepts are: 0 x -intercepts are: $0, -4$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^2 - 4) = \infty$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^3 - 4t) = \lim_{t \rightarrow \pm\infty} t(t^2 - 4) = \pm\infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dy}{dt} = 3t^2 - 4$$

$$\frac{dx}{dt} = 2t$$

$$0 = (\sqrt{3}t - 2)(\sqrt{3}t + 2)$$

$$0 = 2t$$

$$t = \pm \frac{2}{\sqrt{3}} \text{ and } \frac{dx}{dt} \neq 0$$

$$t = 0 \text{ and } \frac{dy}{dt} \neq 0$$

\therefore The curve has horizontal tangents

\therefore The curve has vertical

$$\text{at } \left(-\frac{8}{3}, -\frac{16}{3\sqrt{3}} \right) \text{ and } \left(-\frac{8}{3}, \frac{16}{3\sqrt{3}} \right)$$

tangent at $(-4, 0)$

The Chart

We will use the critical values of t to determine the directions of the curve

t	$-\frac{2}{\sqrt{3}}$	0	$\frac{2}{\sqrt{3}}$
$\frac{dx}{dt}$	-	-	+
$\frac{dy}{dt}$	+	-	-
x	\leftarrow	\leftarrow	\rightarrow
y	\uparrow	\downarrow	\downarrow
curve	\nwarrow	\searrow	\nearrow

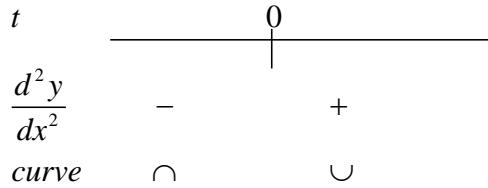
Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 4}{2t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{3t^2 - 4}{2t}\right)}{\frac{dx}{dt}} = \frac{\frac{6t(2t) - 2(3t^2 - 4)}{(2t)^2}}{2t} = \frac{3t^2 + 4}{4t^3}$$

Therefore, the number which appear on the number line is $t = 0$.



3) Intercepts

$$x = 0 \quad y = 0$$

$$0 = t^2 - t \quad 0 = t^3 - 3t$$

$$0 = t(t-1) \quad 0 = t(t-\sqrt{3})(t+\sqrt{3})$$

$$t = 0, 1 \quad t = 0, \pm\sqrt{3}$$

$$\therefore y\text{-intercepts are: } 0, -2 \quad x\text{-intercepts are: } 0, 3 - \sqrt{3}, 3 + \sqrt{3}$$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^2 - t) = \lim_{t \rightarrow \pm\infty} t(t-1) = \infty \quad \sqrt{\quad \quad \quad \quad \quad \quad}$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^3 - 3t) = \lim_{t \rightarrow \pm\infty} t(t^2 - 3) = \pm\infty \quad \sqrt{\quad \quad \quad \quad \quad \quad}$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dy}{dt} = 3t^2 - 3 \quad \frac{dx}{dt} = 2t - 1$$

$$0 = 3(t-1)(t+1) \quad 0 = 2t - 1$$

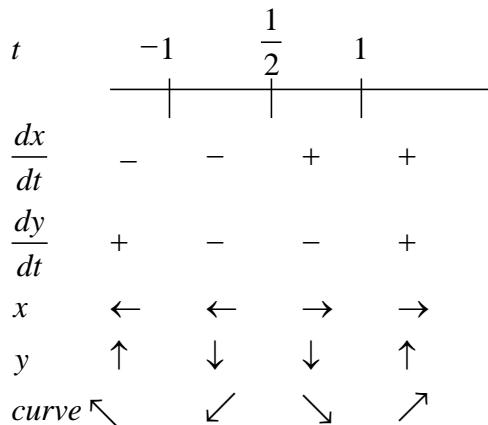
$$t = \pm 1 \text{ and } \frac{dx}{dt} \neq 0 \quad t = \frac{1}{2} \text{ and } \frac{dy}{dt} \neq 0$$

\therefore The curve has horizontal tangents \therefore The curve has vertical

$$\text{at } (0, -2) \text{ and } (2, 2) \quad \text{tangent at } \left(-\frac{1}{4}, -\frac{11}{8}\right)$$

The Chart

We will use the critical values of t to determine the directions of the curve



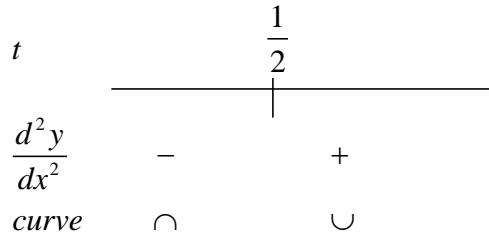
Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 3}{2t - 1}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{3t^2 - 3}{2t - 1}\right)}{\frac{dx}{dt}} = \frac{\frac{6t(2t-1) - 2(3t^2 - 3)}{(2t-1)^2}}{2t-1} = \frac{6(t^2 - t + 1)}{(2t-1)^3}$$

Therefore, the number which appear on the number line is $t = \frac{1}{2}$.



4) Intercepts

$$y = 0 \qquad \qquad \qquad x = 0$$

$$0 = t^2 + t \qquad \qquad \qquad 0 = t^2 + 2t$$

$$0 = t(t+1) \qquad \qquad \qquad 0 = t(t+2)$$

$$t = 0, -1 \qquad \qquad \qquad t = 0, -2$$

$\therefore x$ -intercepts are: $0, -1$ y -intercepts are: $0, 2$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^2 + 2t) = \lim_{t \rightarrow \pm\infty} [t(t+2)] = \infty$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^2 + t) = \lim_{t \rightarrow \pm\infty} t(t+1) = \infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 2t + 2$$

$$0 = 2(t+1)$$

$$t = -1 \text{ and } \frac{dy}{dt} \neq 0$$

\therefore The curve has vertical tangent

$$\text{at } (-1, 0)$$

$$\frac{dy}{dt} = 2t + 1$$

$$0 = 2t + 1$$

$$t = -\frac{1}{2} \text{ and } \frac{dx}{dt} \neq 0$$

\therefore The curve has horizontal

$$\text{tangent at } \left(-\frac{3}{4}, -\frac{1}{2}\right)$$

The Chart

We will use the critical values of t to determine the directions of the curve

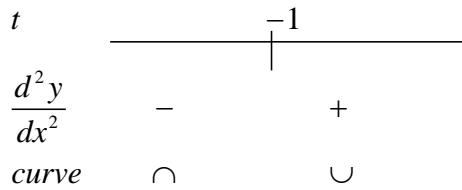
t	— —	-1	$\frac{-1}{2}$
$\frac{dx}{dt}$	—	+	+
$\frac{dy}{dt}$	—	—	+
x	\leftarrow	\rightarrow	\rightarrow
y	\downarrow	\downarrow	\uparrow
$curve$	\swarrow	\searrow	\nearrow

Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t+2} \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{2t+1}{2t+2}\right)}{\frac{dx}{dt}} = \frac{\frac{2(2t+2) - 2(2t+1)}{(2t+2)^2}}{2t+2} \\ &= \frac{2}{(2t+2)^3} \end{aligned}$$

Therefore, the number which appears on the number line is $t = -1$.



5) Intercepts

$$y = 0 \quad x = 0$$

$$0 = t^2 - 2t \quad 0 = t^2$$

$$0 = t(t - 2) \quad 0 = 0$$

$$t = 0, 2 \quad y\text{-intercepts are: } 0$$

$$\therefore x\text{-intercepts are: } 0, 4$$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} t^2 = \infty$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^2 - 2t) = \lim_{t \rightarrow \pm\infty} t(t - 2) = \infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 2t - 2$$

$$0 = 2t \quad 0 = 2t - 2$$

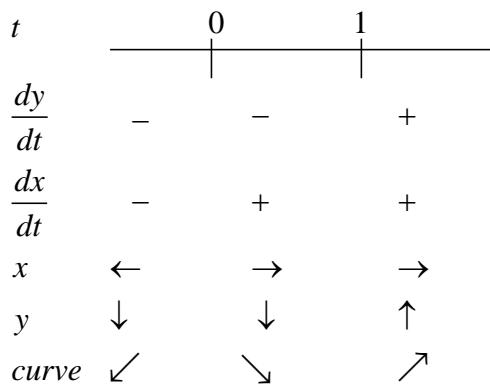
$$t = 0 \text{ and } \frac{dy}{dt} \neq 0 \quad t = 1 \text{ and } \frac{dx}{dt} \neq 0$$

\therefore The curve has vertical tangent
at $(0, 0)$

\therefore The curve has horizontal
tangent at $(1, -1)$

The Chart

We will use the critical values of t to determine the directions of the curve

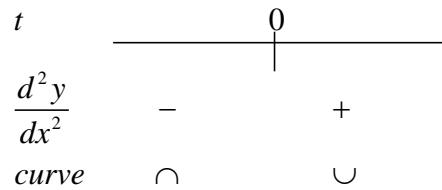


Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-2}{2t} \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{2t-2}{2t}\right)}{\frac{dx}{dt}} = \frac{\frac{2(2t) - 2(2t-2)}{(2t)^2}}{2t} \\ &= \frac{4}{(2t)^3}\end{aligned}$$

Therefore, the number which appears on the number line is $t = 0$.



6) Intercepts

$$y = 0 \qquad \qquad x = 0$$

$$0 = t^2 + t - 2 \qquad \qquad 0 = t^2 + 2t - 1$$

$$0 = (t+2)(t-1) \qquad \qquad t = \frac{-2 \pm \sqrt{2^2 + 4}}{2}$$

$$t = 1, -2 \qquad \qquad t = -1 \pm \sqrt{2}$$

$\therefore x$ -intercepts are: $-1, 2$ y -intercepts are: $\pm\sqrt{2}$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^2 + 2t - 1) = \infty$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^2 + t - 2) = \infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 2t + 2$$

$$0 = 2(t+1)$$

$$t = -1 \text{ and } \frac{dy}{dt} \neq 0$$

\therefore The curve has vertical tangent

$$\text{at } (-2, -2)$$

$$\frac{dy}{dt} = 2t + 1$$

$$0 = 2t + 1$$

$$t = -\frac{1}{2} \text{ and } \frac{dx}{dt} \neq 0$$

\therefore The curve has horizontal

$$\text{tangent at } \left(-\frac{7}{4}, -\frac{9}{4}\right)$$

The Chart

We will use the critical values of t to determine the directions of the curve

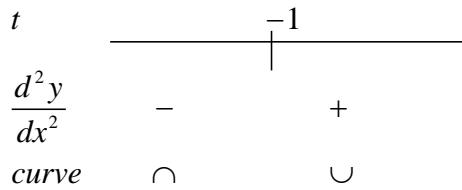
t	— —	-1	$\frac{-1}{2}$
$\frac{dx}{dt}$	—	+	+
$\frac{dy}{dt}$	—	—	+
x	\leftarrow	\rightarrow	\rightarrow
y	\downarrow	\downarrow	\uparrow
$curve$	\swarrow	\searrow	\nearrow

Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t+2} \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{2t+1}{2t+2}\right)}{\frac{dx}{dt}} = \frac{\frac{2(2t+2) - 2(2t+1)}{(2t+2)^2}}{2t+2} \\ &= \frac{2}{(2t+2)^3} \end{aligned}$$

Therefore, the number which appears on the number line is $t = -1$.



7) Intercepts

$$y = 0 \quad x = 0$$

$$0 = t^3 - 3t^2 \quad 0 = t^3 - 3t$$

$$0 = t^2(t-3) \quad 0 = t(t-\sqrt{3})(t+\sqrt{3})$$

$$t = 0, 3 \quad t = 0, \sqrt{3}, -\sqrt{3}$$

$$\therefore x\text{-intercepts are: } 0, 18 \quad y\text{-intercepts are: } 0, \pm 3\sqrt{3} - 9$$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^3 - 3t) = \lim_{t \rightarrow \pm\infty} [t(t^2 - 3)] = \pm\infty \quad \sqrt{\quad \sqrt{\quad}}$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^3 - 3t^2) = \lim_{t \rightarrow \pm\infty} [t^2(t-3)] = \pm\infty \quad \sqrt{\quad}$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 3t^2 - 3 \quad \frac{dy}{dt} = 3t^2 - 6t$$

$$0 = 3(t-1)(t+1) \quad 0 = 3t(t-2)$$

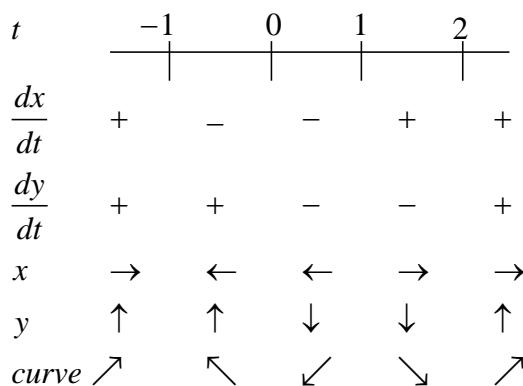
$$t = 1, -1 \text{ and } \frac{dy}{dt} \neq 0 \quad t = 0, 2 \text{ and } \frac{dx}{dt} \neq 0$$

\therefore The curve has vertical tangents at $(-2, -2)$ and $(2, -4)$ \therefore The curve has horizontal tangents at $(0, 0)$ and $(2, -4)$

Therefore, the curve crosses itself at $(2, -4)$ when $t = -1$ and $t = 2$.

The Chart

We will use the critical values of t to determine the directions of the curve

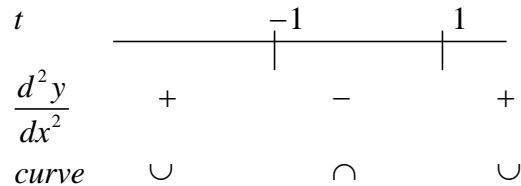


Use the above information, especially the curve directions in the chart to sketch the curve. “sketch”

Check the concavity

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 6t}{3t^2 - 3} = \frac{t^2 - 2t}{t^2 - 1} \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{t^2 - 2t}{t^2 - 1}\right)}{\frac{dx}{dt}} = \frac{(2t-2)(t^2-1) - 2t(t^2-2t)}{(t^2-1)^2} \\ &= \frac{2(t^2-t+1)}{3(t^2-1)^3} \\ &= \frac{2(t^2-t+1)}{3(t-1)^3(t+1)^3}\end{aligned}$$

Therefore, the numbers which appear on the number line are $t = -1$ and 1 .



8) Intercepts

$$y = 0 \qquad \qquad \qquad x = 0$$

$$0 = t^2 - 2t \qquad \qquad \qquad 0 = t^3$$

$$0 = t(t-2) \qquad \qquad \qquad t = 0$$

$$t = 0, 2 \qquad \qquad \qquad y - \text{intercepts are: } 0$$

$\therefore x - \text{intercepts are: } 0, 8$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} t^3 = \pm\infty$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^2 - 2t) = \lim_{t \rightarrow \pm\infty} [t(t-2)] = \infty$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 3t^2$$

$$0 = 3t^2$$

$$t = 0 \text{ and } \frac{dy}{dt} \neq 0$$

\therefore The curve has vertical tangents at $(0, 0)$

$$\frac{dy}{dt} = 2t - 2$$

$$0 = 2(t - 1)$$

$$t = 1 \text{ and } \frac{dx}{dt} \neq 0$$

\therefore The curve has horizontal tangents at $(1, -1)$

The Chart

We will use the critical values of t to determine the directions of the curve

t	0	1	
$\frac{dy}{dt}$	-	-	+
$\frac{dx}{dt}$	+	+	+
x	\rightarrow	\rightarrow	\rightarrow
y	\downarrow	\downarrow	\uparrow
<i>curve</i>	\searrow	\searrow	\nearrow

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-2}{3t^2} \\ \frac{d^2y}{dx^2} &= \frac{d\left(\frac{2t-2}{3t^2}\right)}{dt} = \frac{2(3t^2) - 6t(2t-2)}{(3t^2)^3} = \frac{4-2t}{9t^5} \end{aligned}$$

Therefore, the number which appear on the number line is $t = 0$.

t	0	2
$\frac{d^2y}{dx^2}$	-	+
<i>curve</i>	\cap	\cup

9) Intercepts

$$y = 0$$

$$x = 0$$

$$0 = t^2 - 2t$$

$$0 = t^3 - 3t$$

$$0 = t(t-2)$$

$$0 = t(t-\sqrt{3})(t+\sqrt{3})$$

$$t = 0, 2$$

$$t = 0, \sqrt{3}, -\sqrt{3}$$

$\therefore x$ -intercepts are: 0, 2

y -intercepts are: $0, 3 \pm 2\sqrt{3}$

Beginning and Ending Positions

$$\lim_{t \rightarrow \pm\infty} x = \lim_{t \rightarrow \pm\infty} (t^3 - 3t) = \lim_{t \rightarrow \pm\infty} [t(t^2 - 3)] = \pm\infty \quad \sqrt{\quad}$$

$$\lim_{t \rightarrow \pm\infty} y = \lim_{t \rightarrow \pm\infty} (t^2 - 2t) = \lim_{t \rightarrow \pm\infty} t(t-2) = \infty \quad \sqrt{\quad}$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = 3t^2 - 3$$

$$\frac{dy}{dt} = 2t - 2$$

$$0 = 3(t-1)(t+1)$$

$$0 = 2t - 2$$

$$t = 1, -1 \text{ and } \frac{dy}{dt} \neq 0 \text{ when } t = -1$$

$$t = 1 \text{ and } \frac{dx}{dt} = 0$$

$$\text{But } \frac{dy}{dt} = 0 \text{ when } t = 1$$

What happens when $t=1$?

\therefore The curve has vertical tangents

$$\frac{dy}{dx} = \frac{2t-2}{3t^2-3} \begin{cases} 0 & \text{when } t=1 \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$\text{at } (2, 3)$$

$$[H] \lim_{t \rightarrow 1} \frac{2}{6t} = \frac{1}{3}$$

ie. When $t = 1$ the slope of the tangent approaches $\frac{1}{3}$.

The Chart

We will use the critical values of t to determine the directions of the curve

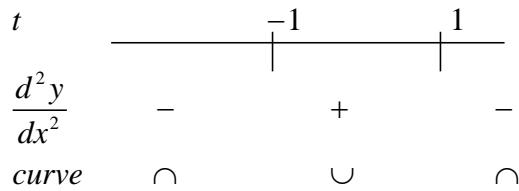
t			
	-1		1
$\frac{dx}{dt}$	+	-	+
$\frac{dy}{dt}$	-	-	+
x	\rightarrow	\leftarrow	\rightarrow
y	\downarrow	\downarrow	\uparrow
<i>curve</i>	\searrow	\swarrow	\nearrow

Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-2}{3t^2-3} \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{2t-2}{3t^2-3}\right)}{\frac{dx}{dt}} = \frac{\frac{2(3t^2-3)-6t(2t-2)}{(3t^2-3)^2}}{3t^2-3} \\ &= \frac{-6(t-1)^2}{27(t^2-1)^3} \\ &= \frac{-6(t-1)^2}{27(t-1)^3(t+1)^3}\end{aligned}$$

Therefore, the numbers which appear on the number line are $t = -1$ and 1 .



10) Intercepts

$$y = 0 \qquad \qquad \qquad x = 0$$

$$0 = 4e^{\frac{t}{2}} \qquad \qquad \qquad 0 = e^t - t$$

$$\emptyset \qquad \qquad \qquad t = e^t$$

\emptyset , since the curves $y = x$ and $y = e^x$ do not intersect.

There are no intercepts.

Beginning and Ending Positions

$$\lim_{t \rightarrow \infty} x = \lim_{t \rightarrow \infty} (e^t - t) = \lim_{t \rightarrow \infty} t \left(\frac{e^t}{t} - 1 \right)$$

But $\lim_{t \rightarrow \infty} \left(\frac{e^t}{t} \right) \left[\begin{matrix} \infty \\ \infty \end{matrix} \right]$

$$[H] \lim_{t \rightarrow \infty} \left(\frac{e^t}{t} \right) = \lim_{t \rightarrow \infty} \left(\frac{e^t}{1} \right) = \infty$$

$$\therefore \lim_{t \rightarrow \infty} x = \lim_{t \rightarrow \infty} t \left(\frac{e^t}{t} - 1 \right) = \infty \cdot \infty = \infty$$

$$\lim_{t \rightarrow -\infty} x = \lim_{t \rightarrow -\infty} (e^t - t) = \lim_{t \rightarrow -\infty} \left(\frac{1}{e^t} + t \right) = 0 + \infty = \infty$$

$$\lim_{t \rightarrow \infty} y = \lim_{t \rightarrow \infty} 4e^{\frac{t}{2}} = \infty \quad \text{and} \quad \lim_{t \rightarrow -\infty} y = \lim_{t \rightarrow -\infty} 4e^{\frac{t}{2}} = 0$$

Coordinates of points of horizontal and vertical tangent lines.

$$\frac{dx}{dt} = e^t - 1$$

$$\frac{dy}{dt} = 4e^{\frac{t}{2}} \left(\frac{1}{2} \right)$$

$$0 = e^t - 1$$

$$0 = 4e^{\frac{t}{2}} \left(\frac{1}{2} \right)$$

$$e^t = 1$$

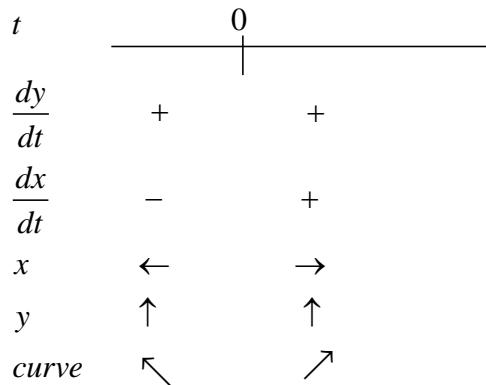
$$\emptyset$$

$$t = 0 \quad \text{and} \quad \frac{dy}{dt} \neq 0$$

\therefore The curve has vertical tangents at $(1, 4)$.

The Chart

We will use the critical values of t to determine the directions of the curve



Use the above information, especially the curve directions in the chart to sketch the curve. "sketch"

Check the concavity

$$\begin{aligned}
\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2e^{\frac{t}{2}}}{e^t - 1} \\
\frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{2e^{\frac{t}{2}}}{e^t - 1}\right)}{\frac{dx}{dt}} = \frac{\frac{e^{\frac{t}{2}}(e^t - 1) - e^t \cdot 2e^{\frac{t}{2}}}{(e^t - 1)^2}}{e^t - 1} = \frac{-e^{\frac{t}{2}}(e^t - 1 - 2e^t)}{(e^t - 1)^3} \\
&= \frac{-e^{\frac{t}{2}}(1 + e^t)}{(e^t - 1)^3}
\end{aligned}$$

\therefore if $e^t - 1 = 0$ then $t = 0$

Therefore, the number which appear on the number line is $t = 0$.

