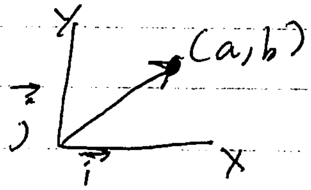
Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$.



$\vec{V} = (a, b)$

$\vec{i} = (1, 0) \qquad \vec{j} = (0, 1)$

$\therefore \vec{V} = a\vec{i} + b\vec{j}$

The length of $\vec{V}$, denoted $\|\vec{V}\|$, is

$\|\vec{V}\| = (a^2+b^2)^{1/2}$.

$\vec{V}$ is a unit vector if $\|\vec{V}\| = 1$.

In 3D:

$\vec{V} = (a, b, c)$

$\vec{i} = (1, 0, 0)$

$\vec{V} = a\vec{i} + b\vec{j} + c\vec{k} \qquad \vec{j} = (0, 1, 0)$

$\vec{k} = (0, 0, 1)$

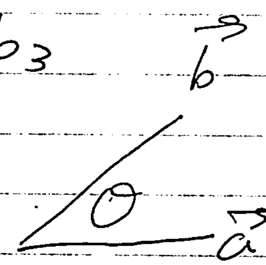
length of $\vec{V}$ in 3D $\quad \|\vec{V}\| = (a^2+b^2+c^2)^{1/2}$

Dot Product. $\quad \vec{a} = (a_1, a_2, a_3)$

$\vec{b} = (b_1, b_2, b_3)$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$= \|\vec{a}\| \, \|\vec{b}\| \cos\theta$



The directional derivative of $f(x,y)$ at (x_0, y_0) is the direction of the unit vector $\vec{u}$ is

$$D_{\vec{u}} f(x_0, y_0) = \lim_{h \to 0} \frac{f((x_0, y_0) + h\vec{u}) - f(x_0, y_0)}{h}$$

let $\vec{u} = (a, b)$ and
$$g(t) = f((x_0, y_0) + t\vec{u}) = f(x_0 + ta, y_0 + tb)$$

$$g'(0) = \lim_{h \to 0} \frac{g(h) - g(0)}{h} = \lim_{h \to 0} [f((x_0, y_0) + h\vec{u}) - f(x_0, y_0)] / h$$
$$= D_{\vec{u}} f(x_0, y_0)$$

$$g'(0) = f_x(x_0, y_0) \frac{d}{dt}(x_0 + ta)\Big|_{t=0}$$
$$+ f_y(x_0, y_0) \frac{d}{dt}(y_0 + tb)\Big|_{t=0}$$
$$= f_x(x_0, y_0)\, a + f_y(x_0, y_0)\, b$$

Example $f(x,y) = x^2 + y^2 + 7y$ at $(1,1)$

Find directional derivative of $f(x,y)$ at $(1,1)$

in the direction $(3,4)$,

$f_x = 2x$ $\quad f_x(1,1) = 2$ $\quad f_y = 7 + 2y$ $\quad f_y(1,1) = 9$

$$D_u f(1,1) = 2 \cdot \frac{u}{\|u\|} \cdot \vec{i} + 9 \cdot \frac{u}{\|u\|} \cdot \vec{j}$$

$$= 2 \cdot \frac{3}{(3^2+4^2)^{1/2}} + 9 \cdot \frac{4}{(3^2+4^2)^{1/2}}$$

$$= \frac{6}{5} + \frac{36}{5} = \frac{42}{5}$$

Ex 2 at $(1,1)$ in the direction towards $(4,5)$.

from $(1,1)$