Recall the equation for a <u>tangent line</u> to a curve:

slope of Tangent $= f'(x_0)$
equation " " $= f(x_0) + f'(x_0)(x - x_0)$
$= T_1(x)$ Taylor polynomial.

Let $P(x_0, y_0, z_0)$ be a point. A plane passing through P has an equation of the form $A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$ where not all $A, B, C \equiv 0$.

If we solve this for z, on a not parallel plane ($c \neq 0$) then we get

$$z = \frac{-A}{C}(x - x_0) + \frac{-B}{C}(y - y_0) + z_0$$

$\frac{-A}{C} =$ slope of the xz slice

$\frac{-B}{C} =$ " " " yz slice

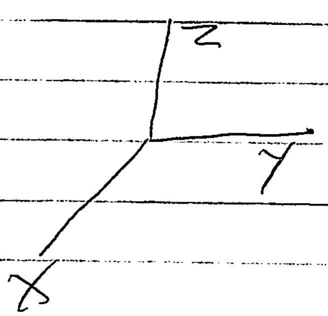
Look near $(x_0, y_0) \in$ domain of f

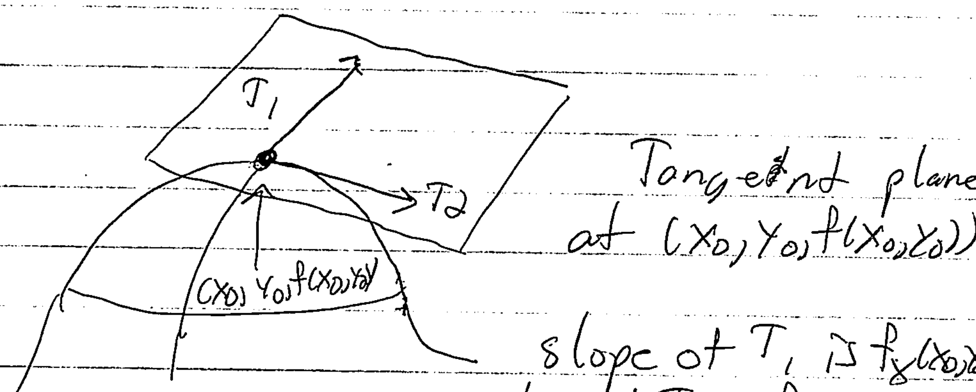
slicing parallel to the xz plane (through $(x_0, y_0, 0)$)

we see $-\frac{A}{C} =$ slope of $g(x) = f(x, y_0)$ at $x_0 = f_x(x_0, y_0)$

slicing parallel to the yz plane $(x_0, 0, 0)$

we see $-\frac{B}{C} =$ slope of $h(y) = f(x_0, y_0)$ at $y_0 = f_y(x_0, y_0)$





slope of T_1 is $f_x(x_0, y_0)$
slope of T_2 is $f_y(x_0, y_0)$

So the equation of the tangent plane is

$$z = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + f(x_0, y_0)$$

Ex Find an equation for the tangent plane through $(1,1,1)$ of $z = 2x^2 - y^3$

$f(x,y) = 2x^2 - y^3$

$f_x(x,y) = 4x$ $\qquad f_y(x,y) = -3y^2$

$f_x(1,1) = 4$ $\qquad f_y(1,1) = -3$

$$z = 4(x-1) + (-3)(y-1) + 1$$
$$= 4x - 4 - 3y + 3 + 1$$
$$= 4x - 3y$$

Sufficient but not necessary Condition for existence of a tangent plane:

If f_x and f_y exist at and near (a,b) and both are continuous at and near (a,b), then $f(x,y)$ has a tangent plane at (a,b)

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

For a function $f(x,y)$ with $(a,b) \in$ Domain,

the linearization of f at (a,b) is the function

$$L(x,y) = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

$f(x,y) \approx L(x,y)$ for (x,y) near (a,b)

Example $f(x,y) = (x^2+y^2)^{1/2} + 3x$
estimate $f(3.01, 4.02)$

Linearze at $(3,4)$

$$f(3,4) = (3^2+4^2)^{1/2} + 3(3) = 5+9 = 14$$

$$f_x = 1/2(x^2+y^2)^{-1/2}(2x) + 3 \qquad f_y = y(x^2+y^2)^{-1/2}$$

$$f_x(3,4) = \frac{3}{5} + 3 = 18/5 \qquad f_y(3,4) = 4/5$$

$$L(x,y) = 18/5 \cdot (x-3) + 4/5(y-4) + 14$$

$$\begin{aligned} f(3.01, 4.02) &\approx L(3.01, 4.02) \\ &= 18/5(.01) + 4/5(.02) + 14 \\ &= 18/5(.01) + 8/5(.01) + 14 \\ &= \frac{26}{5} \cdot \frac{1}{100} + 14 \\ &= 14.052 \end{aligned}$$

$f(x,y,z) = xe^{y}\cos(z)$ at $(1,0,0)=a$

$f_x = e^{y}\cos(z)$ $\quad f_x(a) = 1$

$f_y = xe^{y}\cos(z)$ $\quad f_y(a) = 1$

$f_z = xe^{y}(-\sin(z))$ $\quad f_z(a) = 0$

$$L(x,y,z) = f_x(a)(x-1) + f_y(a)(y-0) + f_z(a)(z-0) + f(1,0,0)$$

$$= (x-1) + y + 1$$

$$= x+y$$

Differential for a function of several variables.

$z = f(x, y)$

$dz := f_x\,dx + f_y\,dy$

Ex $\quad z = f(x, y) = x^2 y + e^y x$

$f_x = 2xy + e^y \qquad f_y = x^2 + e^y x$

$dz = (2xy + e^y)\,dx + (x^2 + e^y x)\,dy$

$w = f(x_1, x_2, \dots, x_n) = x_1^2 + x_2^2 + \dots + x_n^2 \qquad \frac{\partial f}{\partial x_i} = 2x_i$

$dw = \sum_{i=1}^{n} 2x_i\,dx_i$

$\Delta w := f(x_1 + \Delta x_1, x_2 + \Delta x_2, \dots, x_n + \Delta x_n)$

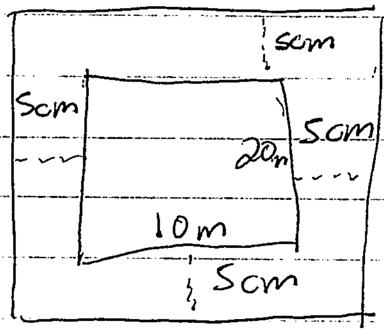
$\qquad - f(x_1, x_2, \dots, x_n)$

increment of w. $\quad \Delta w \approx dw$ for $\Delta x_i = dx_i$

$\Delta w \approx L(x_1 + \Delta x_1, x_2 + \Delta x_2, \dots, x_n + \Delta x_n) -$

$f(x_1, x_2, \dots, x_n) = \sum_{i=1}^{n} f_{x_i} \Delta x_i$

not to scale



Use differentials to approximate the area of the strip.

note 5cm = 0.05m

$A = xy$

$\Delta x = .05 + .05 = .1 = \Delta y$

$dA =$

$dx =$

$dy =$

$dA =$

note exact is

$(10 + .1)(20 + .1) - 10 \cdot 20 = 3.01 m^2$