let $R = [0,2] \times [2,8]$ $n=2, m=2$

Reimann Sums from the point in the upper left corner. $f(x,y) = xy$

Find the R sum.

$R = [1,2] \times [0,\pi]$ find $\iint_R y \sin(xy)\, dA$

Speial Case $f(x,y) = g(x) h(y)$

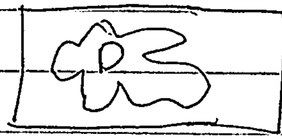
$R = [a,b] \times [c,d]$

$$\iint_R f(x,y)\, dA = \int_a^b \int_c^d g(x)\, h(y)\, dy\, dx$$

$$= \int_a^b g(x) \int_c^d h(y)\, dy\, dx$$

$$= \int_a^b g(x)\, dx \int_c^d h(y)\, dy$$

Integration over general regions



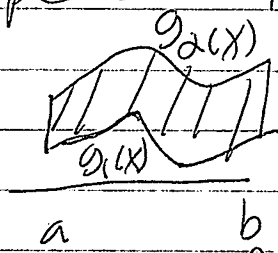
R $\qquad f(x,y)$ with domain D

$$F(x,y) = \begin{cases} f(x,y) & \text{if } (x,y) \in D \\ 0 & \text{otherwise} \end{cases}$$

$$\iint_D f(x,y)\,dA = \iint_R F(x,y)\,dA$$

If the boundary of D consists of a finite union of smooth curves then $F(x,y)$ is integrable.

Type I Regions



D lies between two smooth graphs

$$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

if $d > g_2(x) > g_1(x) > c$ then

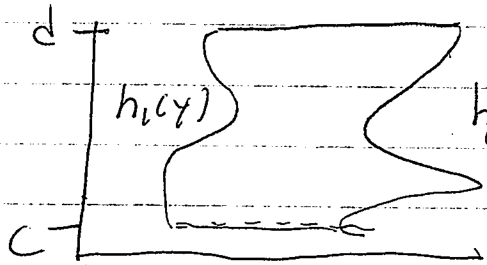
$$\int_c^d F(x,y)\,dy = \int_{g_1(x)}^{g_2(x)} f(x,y) \quad \text{since}$$

$F(x,y) = 0$ if $y > g_2(x)$ or $y < g_1(x)$

$\therefore$ For Type 1

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

Type 2 region

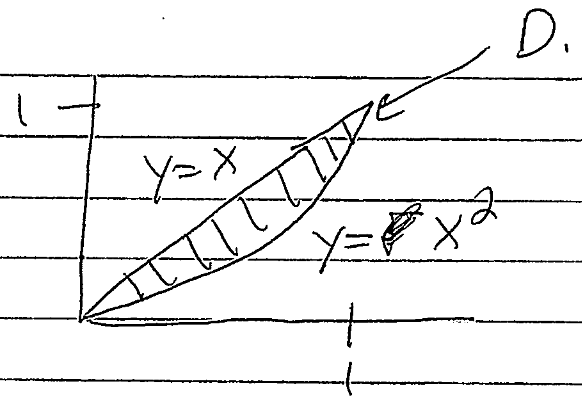


$D = \{(x,y) \mid c \le y \le d,\ h_1(y) \le x \le h_2(y)\}$

$\therefore$ For Type 2

$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

If $f(x,y) \ge 0$ in D, then $\iint_D f(x,y)\,dA$ is the volume of the solid under $z = f(x,y)$ and over the region D.



$z = x^2 + y^2 = f(x,y)$ find $\iint_D f(x,y)\,dA$.

D is the region bounded by

$x=1$, $y=0$, $y=x$

$f(x,y) = e^{x^2}$ find $\iint_D f(x,y)\,dA$