MATH 3GR3 Midterm Test #2

Midterm Test
Duration of test: 50 minutes
McMaster University
November 14, 2023

Instructor: Matt Valeriote

Last Name:________________________________ First Name:________________

Student Number:____

Please answer all five questions. To receive full credit, provide justifications for your solutions. For all questions, write your answers in the answer booklet that has been provided. Please be sure to include your name and student number on all sheets of paper that you hand in.

NOTE: In your solutions you may make use of any theorems or results discussed in the lectures. You may not use other theorems or results, unless you fully justify them. This includes results from the homework assignments.

No aids are allowed.

The number of points each question is worth is indicated in the margin; the maximal number of marks is 25.

Score

Question	1	2	3	4	5	Total
Score						

[6] 1. (a) State Lagrange's Theorem.

(b) Find a subgroup of S_4 that is isomorphic to the group $\mathbb{Z}_4$.

(c) Does S_4 have an element of order 5? Justify your answer to receive credit.

(d) Does S_4 have an element of order 6? Justify your answer to receive credit.

[4] 2. Find an element of the group $\mathbb{Z}_6 \times \mathbb{Z}_4$ of order 12. Does this group have an element of order 8? Justify your answers to receive credit.

[6] 3. Determine which of the following pairs of groups are isomorphic. Justify your answers to receive credit.

(a) $\mathbb{Z}_4$ and $U(8)$.

(b) A_4 and $\mathbb{Z}_{12}$.

(c) $\mathbb{Z}_6 \times \mathbb{Z}_7$ and $\mathbb{Z}_{21} \times \mathbb{Z}_2$.

[4] 4. (a) List the left cosets of the subgroup $\langle(1,2)\rangle$ in the group $\mathbb{Z}_3 \times \mathbb{Z}_3$.

(b) Produce the Cayley table of the factor group $(\mathbb{Z}_3 \times \mathbb{Z}_3)/\langle(1,2)\rangle$.

[5] 5. For each of the following statements, decide if they are **True** or **False**. To receive credit for your answers, you must justify them, by either providing a proof, if the statement is True, or a counter example, if the statement is False.

(a) If the group G is abelian and N is a normal subgroup of G, then the factor group G/N is abelian.

(b) If N is a normal subgroup of the group G such that both N and G/N are abelian, then the group G is abelian.