MATH 3GR3 Midterm #1 Sample Questions

1. (a) Give the definition of a group.

   (b) Let $\mathbb{R}^+ = \{r \in \mathbb{R} : r > 0\}$, the set of all positive real numbers, and for $a, b \in \mathbb{R}^+$, define $a \circ b = \sqrt{ab}$. Is $\mathbb{R}^+$ with the operation $\circ$ a group? Justify your answer.

2. Here is part of an operation table for an operation on a five element set G whose elements are u, v, w, x, y.

$\cdot$	u	v	w	x	y
u	u	v	w		
v					
w		y	x	v	
x	x			y	
y	y	x			u

   Explain why this table can't be filled in so that the operation it defines satisfies the axioms for a group.

3. Produce the Cayley table for $U(12)$, the group of units in $\mathbb{Z}_{12}$.

4. Consider the following element of S_7: $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 6 & 7 & 2 & 4 & 3 \end{pmatrix}$.

   (a) Write μ as a product of disjoint cycles.

   (b) What is the order of μ?

   (c) Determine if μ is an even or an odd permutation.

5. Let n be a natural number with $n > 1$ and suppose that G is a cyclic group of order n. Show that if the natural number m divides into n, then G has a subgroup of order m.