The new deadline is Monday, 27 September by 11:59pm.

MATH 4LT3/6LT3 Assignment #1

Due: Friday, September 24, by 11:59pm.

Upload your solutions to the Avenue to Learn course website.

Detailed instructions will be provided on the course website.

Please read the following statement on collaboration on homework:

Limited collaboration in planning and thinking through solutions to homework problems is allowed, but no collaboration is allowed in writing up solutions. It is permissible to discuss general aspects of the problem sets with other students in the class, but each person should hand in his/her own copy of the solutions. By general aspects I mean you can say things like, "Did you use a diagonalization argument for question 1?" Anything more detailed than this is not acceptable.

Violation of these rules may be grounds for giving no credit for a homework paper and also for serious disciplinary action.

In presenting your solutions, I will be looking for well written, comprehensible answers. Please don't shy away from using complete English sentences to explain your work, and please be careful how you use quantifiers. Every statement you write down should assert something, and should be used somehow to help solve the problem at hand.

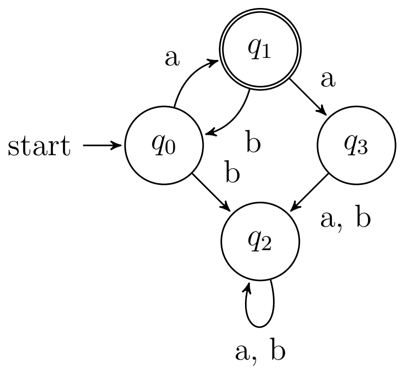
NOTE: You may find it useful to employ the Finite Automata simulator that is linked to from the course homepage (https://www.jflap.org) when working on some of the questions in this assignment. The link will direct you to a simulator that is available as a downloadable java application. You will need to have the appropriate java installation on your computer in order for the simulator to function properly. If you use the application for solving question 2.1.3 please consider sending me via email a saved copy of the automata that you construct, or submitting print outs of their state diagrams.

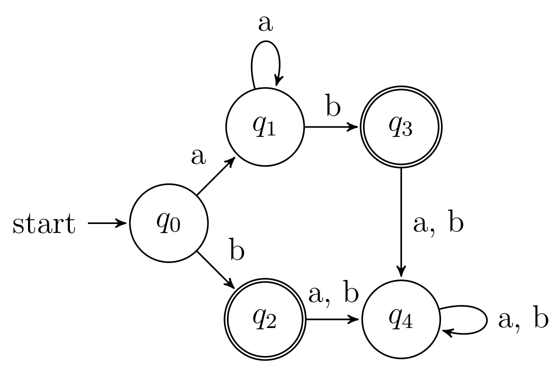
1. Prove the following equalities:

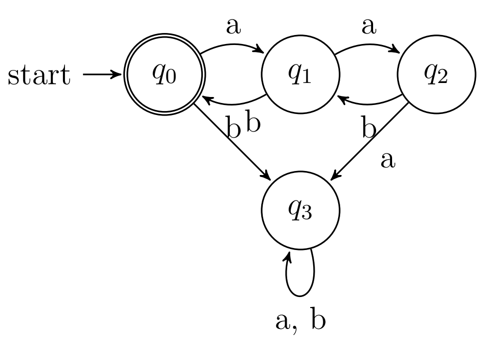
   (a) For any alphabet Σ and any language $L \subseteq \Sigma^*$, $(L^*)^* = L^*$.

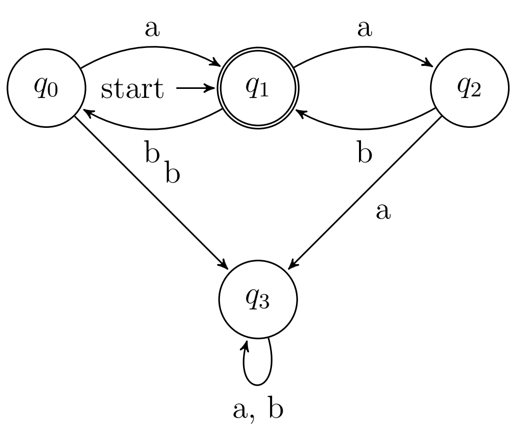
   (b) If a and b are distinct symbols, then $\{a, b\}^* = \{a\}^*(\{b\}\{a\}^*)^*$.

2. For each of the following DFA's, provide an informal description of the languages that they accept.









3. Construct DFA's that accept the following languages:

   (a) $\{w \in \{a,b\}^* \mid$ each a in w is immediately preceded by a $b\}$,

   (b) $\{w \in \{a,b\}^* \mid w$ does not have aa or bb as a substring$\}$,

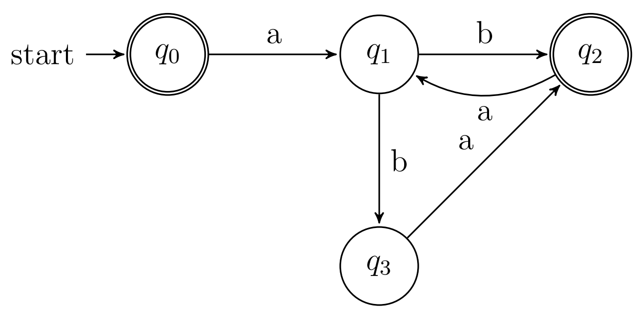
   (c) $\{w \in \{a,b\}^* \mid w$ has an odd number of a's and an even number of b's$\}$,

4. Let $\Sigma = \{a,b\}$. Find regular expressions for the following languages:

   (a) All strings in Σ^* for which the number of a's in it is a multiple of 3.

   (b) All strings in Σ^* that have exactly one occurrence of the substring aaa.

5. Consider the following NFA:



   (a) Determine which of the following strings are accepted by this NFA: ϵ, ab, $abab$, aba, $abaa$.

   (b) Find a regular expression that describes the language accepted by this NFA.

6. Draw state diagrams for NFA's that accept the following languages:

   (a) $(ab)^*(ba)^* \cup aa^*$.

   (b) $((a^*b^*a^*)^*b)^*$.

7. (a) Construct an NFA that accepts the language $(ab \cup aab \cup aba)^*$.

   (b) Convert this NFA into an equivalent DFA using the procedure provided in the proof of Theorem 3.14. **Note**: You should try to find a small NFA for part a), otherwise the size of the DFA that is constructed from it in part b) will be very large.

The following (multi-part) question is for students enrolled in MATH 6LT3. Students in MATH 4LT3 can treat it as a bonus question.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

1. For σ, $\tau \in \Sigma^*$, define $\sigma \equiv \tau$ if for all $\omega \in \Sigma^*$, $\sigma\omega \in L(M)$ if and only if $\tau\omega \in L(M)$. Show that $\equiv$ is an equivalence relation on Σ^*.

2. For σ, $\tau \in \Sigma^*$, define $\sigma \sim \tau$ if $\delta(q_0, \sigma) = \delta(q_0, \tau)$. Show that $\sim$ is an equivalence relation on Σ^* and that $\sim$ is a refinement of $\equiv$. Argue that the number of equivalence classes of $\equiv$ is at most $|Q|$, the number of states of M. Note that we are regarding δ as an extended transition function.

3. For $\sigma \in \Sigma^*$, let $[\sigma]$ denote the equivalence class of strings that are $\equiv$-related to σ. Define M' to be the DFA $(Q', \Sigma, \delta', [\epsilon], F')$, where

   (a) $Q' = \{[\sigma] \mid \sigma \in \Sigma^*\}$,

   (b) $F' = \{[\sigma] \mid \sigma \in L(M)\}$, and

   (c) For $\sigma \in \Sigma^*$ and $a \in \Sigma$, $\delta'([\sigma], a) = [\sigma \cdot a]$.

   Show that δ' is a well-defined function. Then show that $L(M') = L(M)$.

4. Explain why, amongst all of the DFA's that accept the language $L(M)$, M' has the fewest number of states.

5. For the following DFA M, construct the DFA M' using the above definition.