MATH 4LT/6LT3 Assignment #2
Due: Friday, 8 October by 11:59pm.

1. Show that each of the following languages is not regular:
   (a) $L_0 = \{0^n 1^n 2^n \mid n \geq 0\}$.
   (b) $L_1 = \{www \mid w \in \{a, b\}^*\}$.
   (c) $L_2 = \{a^m b^n \mid m \neq n\}$.
   (d) $L_3 = \{x = y + z \mid |x| = |y| + |z| \text{ for } x,\ y,\ z \in \{1\}^*\}$. L_3 is a language over the alphabet $\{1, =, +\}$.
   (e) For Σ an alphabet, L_4 is the set of all regular expressions over Σ.

2. Let $\Sigma = \{a, b\}$ and let L consist of all strings $w \in \Sigma^*$ that contain an equal number of occurrences of the substring ab as it does the substring ba. Show that L is regular. So, $aba \in L$ since it contains one occurrence of ab and one of ba, while $abab$ is not in L.

3. Let Σ be an alphabet and $w \in \Sigma^*$. If $w = a_1 a_2 \dots a_k$ for some $k \geq 0$ and $a_i \in \Sigma$, define the reverse of w to be the string $w^r = a_k a_{k-1} \dots a_2 a_1$. Show that if L is a regular language, then so is $L^r = \{w^r \mid w \in L\}$.

4. Let $T = (\{q_0, q_1, q_2, q_{accept}, q_{reject}\}, \{a\}, \{a, \sqcup\}, \delta, q_0, q_{accept}, q_{reject})$ be the Turing Machine whose transition function is given by the following table:

$Q \times \Gamma$	δ
$(q_0, \sqcup)$	$(q_{accept}, \sqcup, R)$
(q_0, a)	(q_1, a, R)
$(q_1, \sqcup)$	$(q_2, \sqcup, R)$
(q_1, a)	$(q_0, \sqcup, R)$
$(q_2, \sqcup)$	$(q_2, \sqcup, R)$
(q_2, a)	$(q_2, \sqcup, R)$

   (a) Show that T accepts the string $aaaa$ by writing down the sequence of configurations that T occupies, after being started in the initial configuration q_0aaaa.
   (b) Describe the sequence of configurations that T occupies after being started in the configuration q_0aaa. Does T accept aaa?

(c) For $n \geq 0$, carefully describe what T does when started in the configuration $q_0 a^n$.

(d) Describe the language accepted by T.

5. Consider the language L over the alphabet $\{0, 1\}$ consisting of all words w that contain an equal number of 0's and 1's.

   (a) Describe a Turing Machine that accepts L. Your description should describe how your TM operates on an input word w without going into too much detail.

   (b) Draw a state diagram for the TM described in part a). To save time and effort you need not include any transitions that do not matter for the operation of your TM. You may use the JFLAP software for this part of the problem.

do not hand in a solution to this question. It will be part of assignment #3.

6. A common definition of a Turing Machine allows for the machine to not move after reading a symbol, instead of always having to move one cell to the left or to the right. Consider a variant Turing Machine whose transition function may also produce a triple $\delta(q, s) = (q', a', S)$, where the S indicates that the read/write head of the TM does not move.

   Describe how a standard 1-tape Turing machine can be used to simulate the running of one of these variant (but still 1-tape) machines. More precisely, given a specification $T = (Q, \Sigma, \Gamma, \delta, q_s, q_{accept}, q_{reject})$ of a variant TM, give a definition of a standard TM T' which essentially does the same things that T does.

The following questions are for students enrolled in MATH 6LT3. Students in MATH 4LT3 can treat them as bonus questions.

B1 Let L be a regular language over an alphabet Σ. Show that there is some TM that decides L.

B2 For L_1 and L_2 languages over the alphabet Σ, define $L_1 \wr L_2$ to be the language

$$\{w \in \Sigma^* \mid w = a_1 b_1 a_2 b_2 \dots a_k b_k \text{ for some } k \geq 0,\ a_1 a_2 \dots a_k \in L_1$$
$$\text{and } b_1 b_2 \dots b_k \in L_2\}.$$

Prove that if L_1 and L_2 are regular then so is $L_1 \wr L_2$.