MATH 4LT3/6LT3 Midterm Test Solutions

Midterm Test
Duration of test: 50 minutes
McMaster University
October 24, 2025

Instructor: Matt Valeriote

Last Name:________________________________ First Name:________________

Student Number:____

This test consists of **four** pages and **four** questions. Please answer all four questions. For all questions, write your answers in the answer booklet that has been provided. Please be sure to include your name and student number on all sheets of paper that you hand in.

NOTE: In your solutions you may make use of any theorems or results discussed in the lectures. You may not use other theorems or results, unless you fully justify them. This includes results from the homework assignments, unless otherwise stated.

No aids are allowed.

Each question is worth 5 points; the maximal number of marks is 20.

Score

Question	1	2	3	4	Total
Score					

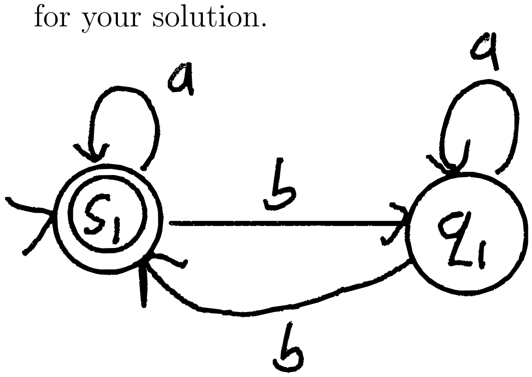
1. Consider the following language over the alphabet $\Sigma = \{a, b\}$:

$$L = \{w \in \Sigma^* \mid \#_b(w) \text{ is even } \textbf{or } \#_a(w) = 2\}.$$

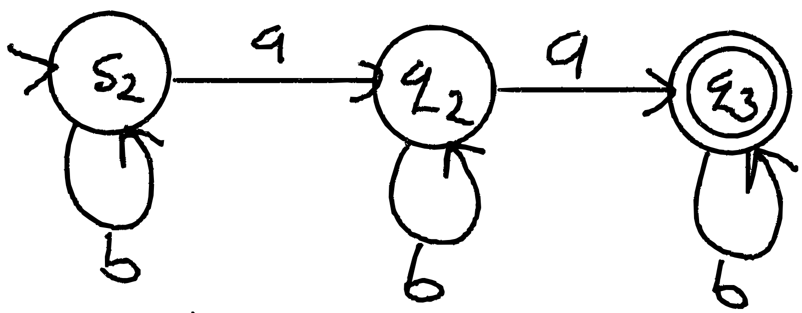
Recall that for a string v and symbol x, $\#_x(v)$ is the number of occurrences of the symbol x in the string v.

So the strings aaa, bb, $abbbaab$, $babbbab$, $abaaba$ all belong to L and the strings $abaa$ and $babb$ do not. The empty string belongs to L.

Show that L is a **regular language** by providing an NFA M with $L(M) = L$. Your solution should be in the form of a diagram for the NFA M that you construct. You do not need to provide a justification for your solution.



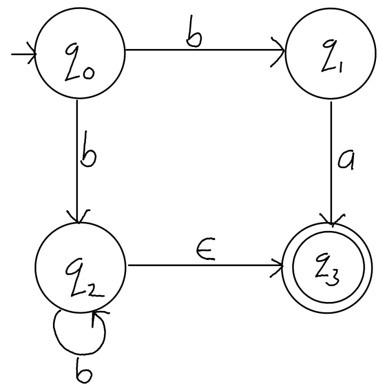
This part of M accepts strings that have an odd number of b's.



this part accepts strings that have exactly 2 a's.

This NFA has 2 initial states, s_1 and s_2, and 2 accept states, s_1 and q_3

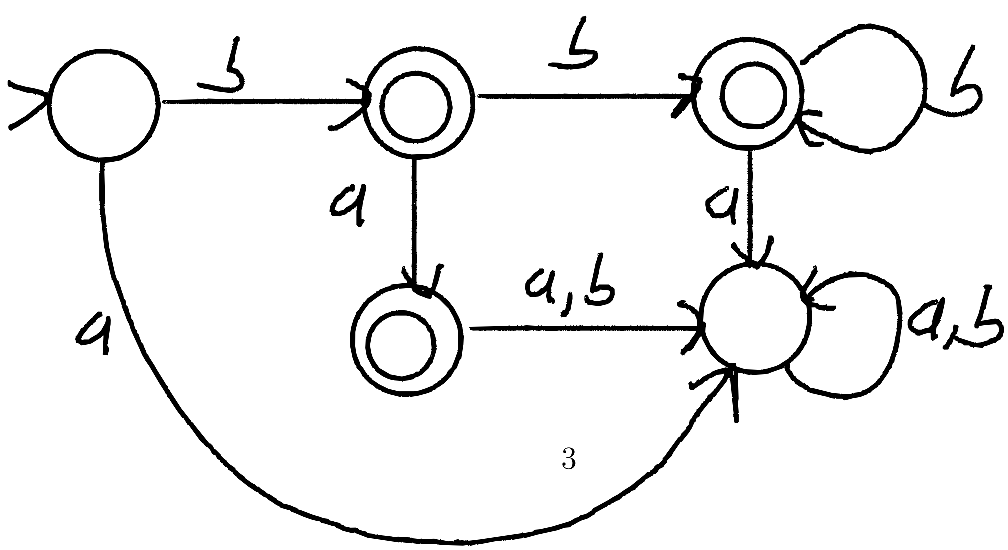
2. Consider the following diagram for the ϵ-NFA M over the alphabet $\{a, b\}$:



(a) Describe the language, $L(M)$, that this NFA accepts. You do not need to provide a justification for your answer.

$L(M) = \{ba\} \cup \{b^n \mid n > 0\}$.
So, $L(M)$ consists of the string ba and all strings of b's of length at least 1.

(b) Construct a DFA M' with $L(M') = L(M)$. Your solution should be in the form of diagram for the DFA M'. You do not need to provide a justification for your solution. You do not need to use the subset construction in your solution, but you can if you wish.



For $1 \le i < j$, let $w = b^i$.
Then $a^{3i}w \in L$ and $a^{3j}w \notin L$.

3. Consider the following language L over the alphabet $\Sigma = \{a, b\}$:

$$L = \{w \in \Sigma^* \mid \#_a(w) = 3 \times (\#_b(w))\}.$$

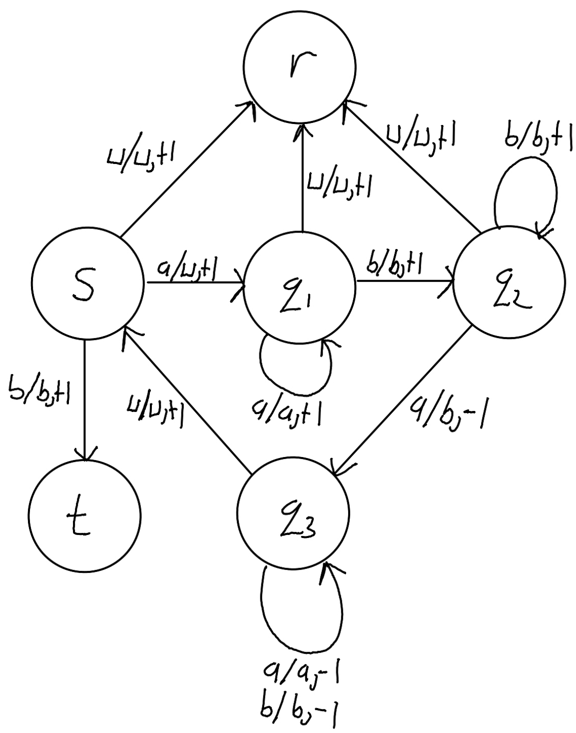
So the strings *abaa* and *baabaaaa* both belong to L but the strings *aaa*, *baab* do not. The empty string also belongs to L.

Prove that the language L is **not regular**.

Let $S = \{a^{3n} \mid n \ge 1\}$. Then S is an infinite pairwise separable set for L, and so L is not regular.

4. The following is a diagram for the **Deterministic Turing Machine** (DTM) $T = (Q, \Sigma, \Gamma, s, t, r, \delta)$ where

- $Q = \{s, t, r, q_1, q_2, q_3\}$,
- $\Sigma = \{a, b\}$, and
- $\Gamma = \{a, b, \sqcup\}$.



$[s, ababb\sqcup\ldots, 1]$

(a) On input string *ababb*, the DTM T begins in the initial configuration $[s, ababb, 1]$. Write down the next three configurations of the computation of T on input *ababb*.

$$[s, ababb\sqcup\ldots, 1] \vdash_T [q_1, \sqcup\underline{b}abb\sqcup\ldots, 2] \vdash_T [q_2, \sqcup b\underline{a}bb\sqcup\ldots, 3] \vdash_T [q_3, \sqcup\underline{b}bbb\sqcup\ldots, 2]$$

(b) Which of the following strings are accepted by T: *ababb*, *aababb*, *bbbaa*? You do not need to provide a justification of your solution.

ababb and bbbaa are accepted, aababb is rejected.

(c) Give a brief description of $L(T)$, the set of strings over Σ that T accepts. You do not need to provide a justification of your solution.

L(T) consists of all strings w that have at least one occurrence of b, and the number of occurrences of a's to the right of the first occurrence of b's is at least as big as the number of occurrences of a's to the left of the first b.

So $L(T) = \{ ubv \mid \#_a(u) \leq \#_a(v) \}$