

**SHORT DESCRIPTION OF RESEARCH PAPERS,
BOOKS AND LECTURE NOTES 2005-2009**

1. Weighted inequalities for singular integrals

- (1) M. LACEY, E. SAWYER, AND I. URIARTE-TUERO, *Astala's conjecture on distortion of Hausdorff measure under quasiconformal maps*, to appear in Acta. Math.
- (2) M. LACEY, E. SAWYER, AND I. URIARTE-TUERO, *A characterization of two weight norm inequalities for maximal singular integrals*.
- (3) M. LACEY, E. SAWYER, AND I. URIARTE-TUERO, *A characterization of the two weight norm inequality for the Hilbert transform*.

The third paper answers the old question of when the Hilbert transform is bounded from one weighted L^2 space to another. The proof uses an energy functional in the machinery developed for nonhomogeneous analysis by Nazarov, Treil and Volberg. The second paper comes close to answering an old question regarding two weight L^p inequalities for *maximal* singular integrals. An application (subsequently simplified) was given in the first paper to solve an open problem of K. Astala dealing with the absolute continuity of Hausdorff measures under the application of quasiconformal maps. The two weight problems for maximal singular integrals, and singular integrals themselves (not maximal truncations) when $p \neq 2$, remain open.

2. The corona problem

- (1) S. COSTEA, E.SAWYER AND B. WICK, *The corona theorem for the Drury-Arveson Hardy space and other Besov-Sobolev spaces on the unit ball in C^n* .
- (2) S. COSTEA, E.SAWYER AND B. WICK, *BMO estimates for the $H^\infty(\mathbb{B}_n)$ corona problem*.

In the first paper, the corona question for the multiplier algebra of the Drury-Arveson Hardy space is settled. The analogous question for the multiplier algebra H^∞ of the classical Hardy space still remains open after almost half a century, but the second paper obtains improved *BMO* estimates for this classical problem. Techniques used include

explicit formulas of Charpentier type for solutions to the $\bar{\partial}$ problem with higher degree forms, the algebra of rogue terms and new norms for Besov-Sobolev spaces.

3. Function theory on the disk and ball

- (1) ARCOZZI, NICOLA; ROCHBERG, RICHARD; SAWYER, ERIC AND B. WICK, *Bilinear forms on the Dirichlet space*, to appear in Analysis and PDE.
- (2) ARCOZZI, NICOLA; ROCHBERG, RICHARD; SAWYER, ERIC, *Carleson measures for the Drury-Arveson Hardy space and other Besov-Sobolev spaces on complex balls*, Adv. Math. **218** (2008), no. 4, 1107–1180.
- (3) ARCOZZI, NICOLA; ROCHBERG, RICHARD; SAWYER, ERIC, *Carleson measures and interpolating sequences for Besov spaces on complex balls*, Mem. Amer. Math. Soc. **182** (2006) no. 859, vi+163 pp.

In the first paper, an old conjecture of Rochberg on Hankel forms was settled for the classical Dirichlet space. The second paper characterizes Carleson measures for the Drury-Arveson Hardy space, the generalization of the classical Hardy space on the disk that is appropriate for multivariate operator theory. The third paper characterizes interpolating sequences for the Dirichlet space and other Besov spaces on the unit ball in \mathbb{C}^n . A key new technique used here is the concept of a ‘holomorphic’ tree.

4. The Monge-Ampere equation

- (1) RIOS, CRISTIAN; SAWYER, ERIC T.; WHEEDEN, RICHARD L., *Regularity of subelliptic Monge-Ampère equations*, Adv. Math. **217** (2008), no. 3, 967-1026.
- (2) GUAN, PENGFEI; SAWYER, ERIC, *Regularity of subelliptic Monge-Ampère equations in the plane*, Trans. Amer. Math. Soc. **361** (2009), no. 9, 4581–4591.

The first paper generalizes two dimensional work of Guan to obtain a near optimal regularity theorem for a degenerate Monge-Ampere equation in dimension greater than two. Namely that C^2 solutions u to an equation of Monge-Ampère type,

$$(4.1) \quad \det D^2u = k(x, u, Du), \quad x \in \Omega \subset \mathbb{R}^n,$$

(with certain restrictions) are necessarily smooth if at the finite type zeroes of k , the points of degeneracy, $n - 1$ of the principal curvatures of the solution u are bounded away from zero (fewer than $n - 1$

nonvanishing principal curvatures do not suffice). The proof uses a new application of Calabi's identity, the partial Legendre transform and a perturbation argument using the Stein-Rothschild lifting of vector fields. The Transactions paper completed a program begun by Guan, by showing that for a finite type degeneracy, the nonvanishing of the mean curvature suffices for smoothness of $C^{1,1}$ solutions u to (4.1) in the plane. Another solution to this problem was obtained shortly after using completely different and more powerful techniques by Daskalopoulos and Savin. Still open is the question of whether or not the initial regularity of solutions can be relaxed to the optimal assumption $u \in C^{1,1}$.

5. Degenerate elliptic equations

- (1) SAWYER, ERIC T.; WHEEDEN, RICHARD L., *Hölder continuity of weak solutions to subelliptic equations with rough coefficients*, Mem. Amer. Math. Soc. **180** (2006) no. 847, x+157 pp.
- (2) SAWYER, ERIC T.; WHEEDEN, RICHARD L., *Degenerate Sobolev spaces and regularity of subelliptic equations*, Trans. A. M. S., article electronically published on October 30, 2009.

The first paper obtains an analogue of the theorem of Fefferman and Phong on smooth vector fields. Roughly speaking, *classical* weak solutions of equations involving a rough degenerate operator $L = \nabla' B(x) \nabla$, are Hölder continuous for some positive exponent provided an ε -comparability condition holds, there is a doubling condition on the control balls associated with B , and certain Poincaré and Sobolev inequalities hold relating the subunit metric to the degenerate form $\xi' B(x) \xi$. Moreover, apart from the doubling condition, these conditions are almost necessary. There is also an analogue of Hörmander's commutation theorem on smooth vector fields for rough diagonal vector fields involving a new condition, the flag condition, which is in fact equivalent to Hörmander's commutation condition when the vector fields are analytic. The Transactions paper extended these results to hold for the most *general* notion of weak solution associated with a degenerate quadratic form $\xi' B(x) \xi$. This required a new development of the theory of degenerate Sobolev spaces of both H and W type.

6. OTHER BOOKS AND PAPERS

SAWYER, E., *Function Theory: Interpolation and Corona problems*, Fields Institute Monograph series **25**, American Math. Soc. 2009, ix + 203 pages.

RIOS, C. AND SAWYER, E., *Smoothness of radial solutions to the Monge-Ampère equation*, Proc. Amer. Math. Soc. **137** (2009), no. 4, 1373–1379.

ARCOZZI, NICOLA; ROCHBERG, RICHARD; SAWYER, ERIC, *The diameter space, a restriction of the Drury-Arveson-Hardy space, Function spaces* 21-42, Contemp. Math., **435** (2007), Amer. Math. Soc., Providence, RI.

IOSEVICH, ALEXANDER; SAWYER, ERIC T.; SEEGER, ANDREAS, *Mean lattice point discrepancy bounds. II. Convex domains in the plane*, J. Anal. Math. **101** (2007), 25-63.

SAWYER, ERIC T.; WHEEDEN, RICHARD L., *Regularity of degenerate Monge-Ampère and prescribed Gaussian curvature equations in two dimensions*, Potential Anal. **24** (2006), no. 3 267-301.

SAWYER, ERIC T.; WHEEDEN, RICHARD L., *A priori estimates for quasilinear equations related to the Monge-Ampère equation in two dimensions*, J. Anal. Math. **97** (2005), 257-316.

RIOS, CRISTIAN; SAWYER, ERIC T.; WHEEDEN, RICHARD L., *A higher-dimensional partial Legendre transform, and regularity of degenerate Monge-Ampère equations*, Adv. Math. **193** (2005) no. 2, 373-415.

ARCOZZI, N.; ROCHBERG, R.; SAWYER, E., *Some problems on Carleson measures for Besov-Sobolev spaces*, Topics in complex analysis and operator theory, 141–148, Univ. Málaga, Málaga, 2007.

ARCOZZI, NICOLA; ROCHBERG, RICHARD; SAWYER, ERIC, *The characterization of the Carleson measures for analytic Besov spaces: a simple proof*, Complex and harmonic analysis, 167–177, DEStech Publ., Inc., Lancaster, PA, 2007.

7. LECTURE NOTES

SAWYER, ERIC, *Lecture Notes in Real Analysis 2009*

SAWYER, ERIC, *Lecture Notes in Complex Analysis 2009*

SAWYER, ERIC, *Functional Analysis and Applications 2006*