ASSIGNMENT 15.5

1.(a) $$T_3(x) = \underbrace{\underbrace{f(4) + f'(4)(x-4)}_{T_1(x) \text{ or } L_4(x)} + \frac{f''(4)}{\underset{=2}{2!}}(x-4)^2}_{T_2(x)} + \frac{f'''(4)}{\underset{=6}{3!}}(x-4)^3$$

$f, f', \dots$	at $a=4$
$f = \sqrt{x}$	$f(4) = 2$
$f' = \frac{1}{2\sqrt{x}}$	$f'(4) = \frac{1}{4}$
$f'' = -\frac{1}{4\sqrt{x^3}}$	$f''(4) = -\frac{1}{4\sqrt{64}} = -\frac{1}{32}$
$f''' = \frac{3}{8}\frac{1}{\sqrt{x^5}}$	$f'''(4) = \frac{3}{8\sqrt{4^5}} = \frac{3}{256}$

$$f' = \frac{1}{2}x^{-1/2}$$
$$\to f'' = -\frac{1}{4}x^{-3/2} = -\frac{1}{4\sqrt{x^3}}$$
$$f''' = \left(-\frac{1}{4}\right)\left(-\frac{3}{2}\right)x^{-5/2} = \frac{3}{8}\frac{1}{\sqrt{x^5}}$$

so:

$$T_1(x) = 2 + \frac{1}{4}(x-4)$$

$$T_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32 \cdot 2}(x-4)^2$$
$$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$$

$$T_3(x) = T_2(x) + \frac{3/256}{6}(x-4)^3$$
$$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$$

(b) $\sqrt{4.5}$ using T_1 ~ $T_1(4.5) = 2 + \frac{1}{4}(4.5-4) = \underline{2.12}5$

T_2 .. $T_2(4.5) = 2 + \frac{1}{4}(0.5) - \frac{1}{64}(0.5)^2$ correct

$= \underline{2.121}0937$
correct

$$T_3 \ldots\ T_3(4{,}5) = 2 + \frac{1}{4}(0{,}5) - \frac{1}{64}(0{,}5)^2 + \frac{1}{512}(0{,}5)^3$$
$$= \underbrace{2{,}12133}_{\text{correct}}7891$$

2.(a) $L_1(x) = f(1) + f'(1)(x-1) = 2 + (1)(x-1) = x+1$

f, f'	at $a=1$
$f = x^2 + \frac{1}{x}$	$f(1) = 2$
$f' = 2x - \frac{1}{x^2}$	$f'(1) = 1$

$$f(1{,}4) \approx L_1(1{,}4) = 1{,}4 + 1 = \underline{\underline{2{,}4}}$$

(b) $L_2(x) = f(2) + f'(2)(x-2) = \frac{9}{2} + \frac{15}{4}(x-2) = \frac{15}{4}x - 3$

f, f'	at $a=2$
$f = x^2 + \frac{1}{x}$	$f(2) = 4 + \frac{1}{2} = \frac{9}{2}$
$f' = 2x - \frac{1}{x^2}$	$f'(2) = 4 - \frac{1}{4} = \frac{15}{4}$

$$f(1{,}4) \approx L_2(1{,}4) = \frac{15}{4}(1{,}4) - 3 = \underline{\underline{2{,}25}}$$

(c) $\left.\begin{array}{l} x=1 \longrightarrow f(1) = 2 \\ x=2 \longrightarrow f(2) = 9/2 \end{array}\right\}$ slope $= \dfrac{\frac{9}{2} - 2}{2-1} = \frac{5}{2}$

$$y - 2 = \frac{5}{2}(x-1)$$

ie $y = S(x) = 2 + \frac{5}{2}(x-1) = \frac{5}{2}x - \frac{1}{2}$

$$f(1{,}4) \approx S(1{,}4) = \frac{5}{2}(1{,}4) - \frac{1}{2} = \underline{\underline{3}}$$

(d) $T_2(x) = f(1) + f'(1)(x-1) + \underbrace{\frac{f''(1)}{2!}(x-1)^2}_{\frac{4}{2}(x-1)^2}$

$f(1) \to 2$, $f'(1) \to 1$ from part (a)

$$f'' = 2 + \frac{2}{x^3} \to f''(1) = 4$$

so $T_2(x) = 2 + (x-1) + 2(x-1)^2$

$$f(1{,}4) \approx T_2(1{,}4) = 2 + (0{,}4) + 2(0{,}4)^2 = \underline{\underline{2{,}72}}$$

(e) $T_2(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2$

$\underbrace{f(2) \to 9/2 \quad f'(2) \to 15/4}_{\text{from part (b)}}$ $\quad f''(2) \to 9/4$

$$f'' = 2 + \frac{2}{x^3} \longrightarrow f''(2) = 2 + \frac{2}{8} = \frac{9}{4}$$

so $T_2(x) = \frac{9}{2} + \frac{15}{4}(x-2) + \frac{9}{8}(x-2)^2$

$$f(1{,}4) \approx T_2(1{,}4) = \frac{9}{2} + \frac{15}{4}(-0{,}6) + \frac{9}{8}(-0{,}6)^2$$

$$= \underline{\underline{2{,}655}}$$

compare (a)-(e) with a precise value

$$f(1{,}4) = 2{,}674285714$$

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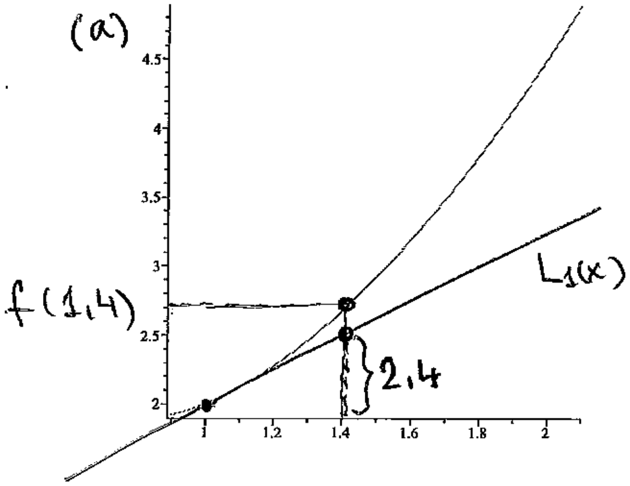
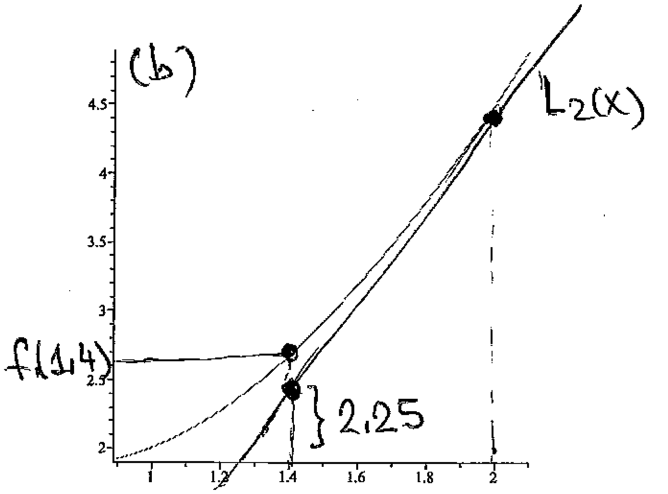
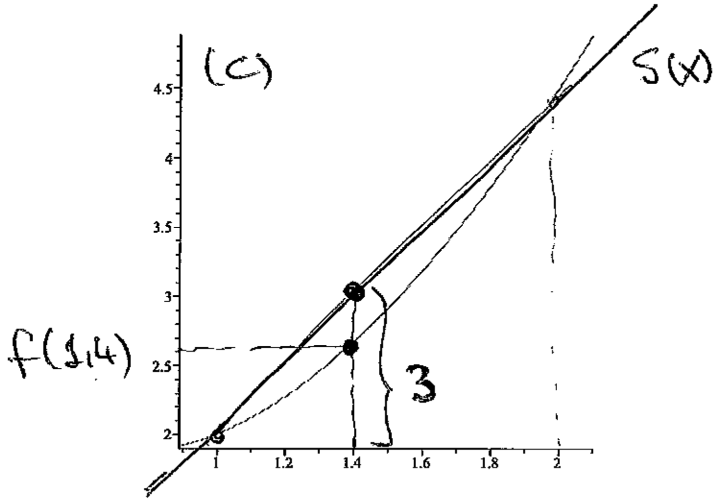
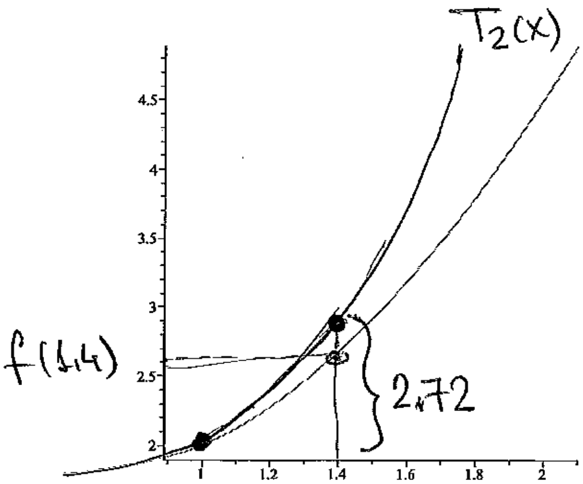
f(1,4) = 2,674285714

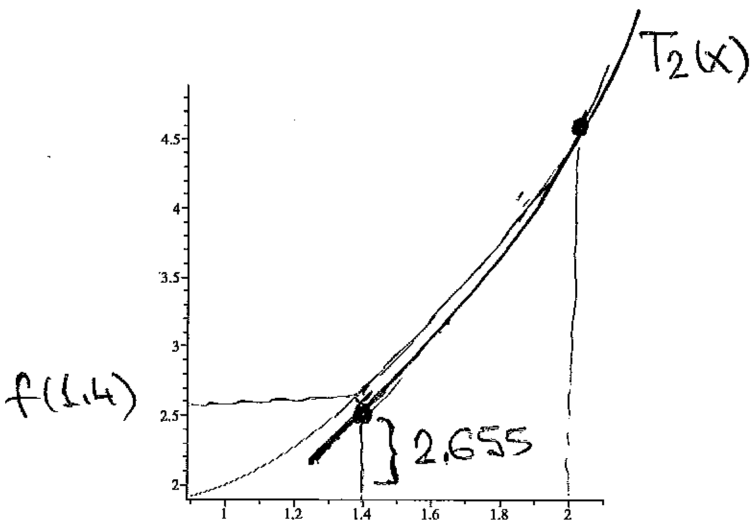
(f) Sketch each approximation (a)-(e).











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3.(a) $\lim_{x\to\infty} x^2 e^{-1{,}3x} = \lim_{x\to\infty} \frac{x^2}{e^{1{,}3x}} = \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{2x}{1{,}3\,e^{1{,}3x}} = \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{2}{(1{,}3)(1{,}3)\,e^{1{,}3x}} = \frac{2}{\infty} = 0$

(b) $\lim_{x\to 0+} (4x)^{5x} = 0^0 = \lim_{x\to 0+} e^{\ln(4x)^{5x}}$

$= \lim_{x\to 0+} e^{5x\cdot\ln(4x)} = e^0 = 1$

$\lim_{x\to 0+} 5x\cdot\ln(4x) = 5\cdot\lim_{x\to 0+} \frac{\ln(4x)}{\frac{1}{x}}$

$\overset{LH}{=} 5\cdot\lim_{x\to 0+} \frac{\frac{1}{\cancel{4}x}\cdot\cancel{4}}{-\frac{1}{x^2}} = 5\cdot\lim_{x\to 0+}(-x) = 0$

(c) $\lim_{x\to\infty} \frac{\ln(\ln x)}{x^2} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{2x}$

$= \lim_{x\to\infty} \frac{1}{2x^2\ln x} = \frac{1}{\infty} = 0$

(d) $\lim_{x\to(\frac{\pi}{2})^-} (\sec x - \tan x) = \infty - \infty$

$= \lim_{x\to(\frac{\pi}{2})^-} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) = \lim_{x\to(\frac{\pi}{2})^-} \frac{1-\sin x}{\cos x} = \frac{0}{0}$

$$\overset{LH}{=} \lim_{x\to(\frac{\pi}{2})^-} \frac{-\cos x}{-\sin x} = \frac{0}{-1} = 0$$

(e) $$\lim_{x\to\infty} \frac{e^x+4x}{x^2+3e^x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x+4}{2x+3e^x}$$

$$= \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{2+3e^x} = \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{3e^x} = \frac{1}{3}$$

(f) $$\lim_{c\to\infty} \frac{4c}{2+\ln(1+c)} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{c\to\infty} \frac{4}{\frac{1}{1+c}\cdot 1}$$

$$= \lim_{c\to\infty} 4(1+c) = \infty$$