ASSIGNMENT 15

1(a) The value of the function $f(x)$, which is larger than the values of $f(x)$ nearby; ie, if $f(c) \geqslant f(x)$ for all x near c then $f(c)$ is a relative max. of f

(b) in both cases, $f(c) \geqslant f(x)$

relative ... for x near c

absolute ... for all x in the domain of f

(c) If a function has an extreme value, then it must occur at a critical point

(d) $\boxed{\text{No.}}$ $f(x)=x^3$ satisfies $f'(0)=0$, but $f(x)$ has no extreme value at $x=0$.

(e) To test a critical point $x=c$ we can use increasing/decreasing argument

if f increases to the left of c and decreases to the right of c, then f has a local (rel.) max. at c

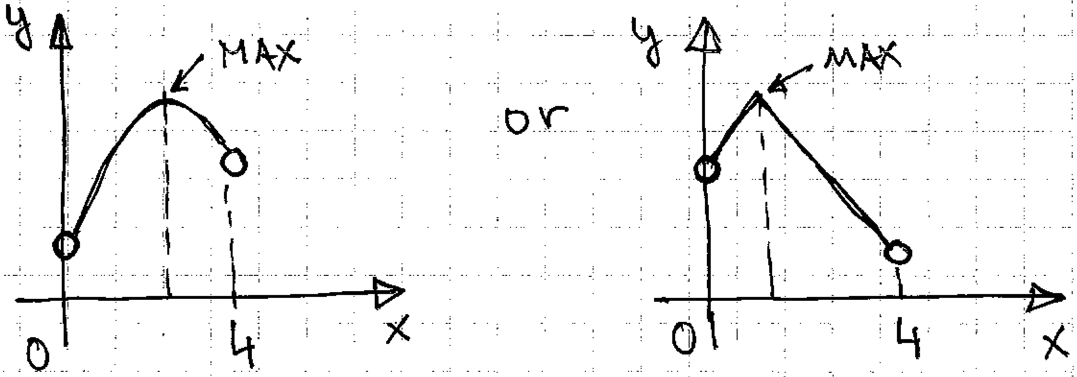
switch increases $\leftrightarrow$ decreases for local minimum

(f) $f(x)=|x|$ at $x=0$ $\longrightarrow$ $f'(0)$ does not exist,

so $f''(0)$ cannot exist either

2. (a) if f is continuous on a closed interval $[a,b]$ then f has an absolute max. and an absolute min. in $[a,b]$

(b)



3.
$$R = K \cdot \frac{1}{d^4}$$

if $d \to 0.8d$ (20% reduction) then

$$R = K \cdot \frac{1}{(0.8d)^4} = \frac{1}{0.8^4} \cdot K \cdot \frac{1}{d^4}$$

$$\approx 2.44 \cdot \text{original resistance}$$

4. (a) $f(x) = x^3 - x + 2$

$f'(x) = 3x^2 - 1 = 0, \quad x^2 = \frac{1}{3} \rightarrow \underline{\underline{x = \pm\sqrt{\frac{1}{3}}}}$

(b) $f(x) = x^3 + x + 2$

$f'(x) = 3x^2 + 1 = 0, \quad \underbrace{x^2 = -\frac{1}{3}}_{\text{no solutions}} \rightarrow \underline{\underline{\text{no c.p.'s}}}$

(c) $f(x) = x \ln x$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 0$

$\ln x = -1 \rightarrow \underline{\underline{x = e^{-1}}}$

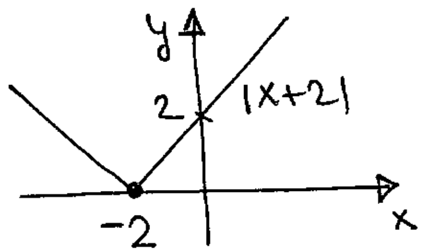
(d) $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x = 0 \rightarrow \sin x = \cos x$

(divide by $\cos x$) $\quad \tan x = 1$

so $\quad \underline{\underline{x = \frac{\pi}{4} + \pi k}}$

(e) $f(x) = |x+2|$



$f'(x) = 0$... no such points (slope is either 1 or -1)

$f'(x)$ dne at $x = -2$

answer: $\underline{\underline{x = -2}}$

(f) $f(x) = x e^{3x}$

$$f'(x) = e^{3x} + x e^{3x} \cdot 3 = e^{3x} \cdot (1+3x) = 0$$

$$\begin{cases} \rightarrow 1+3x=0 \quad x=-1/3 \\ \rightarrow e^{3x}=0 \quad \text{no solutions for } x \end{cases}$$

answer $\underline{\underline{x = -1/3}}$

5. $f'(x) = 4x^3 - 4 = 0 \rightarrow 4(x^3-1) = 0$

so $x^3 = 1$, $x = 1$

x	$f(x) = x^4 - 4x + 3$
1	0 ← abs. min. at $x=1$
0	3
2	11 ← abs. max. at $x=2$

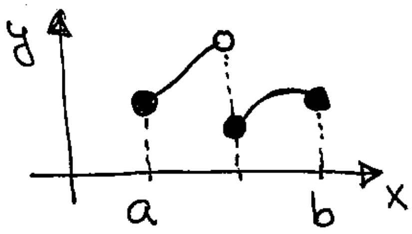
6. $$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2} = 0$$

$$\rightarrow 1 - \ln x = 0, \quad \ln x = 1, \quad x = e^1 = e$$

x	$f(x) = \frac{\ln x}{x}$
e	$\frac{\ln e}{e} = \frac{1}{e} = 0.367$ ← abs. max.
1	$\frac{\ln 1}{1} = \frac{0}{1} = 0$ ← abs. min.
3	$\frac{\ln 3}{3} = 0.366$

7. No. It is true only if the function is continuous on $[a,b]$.

For instance, the function $f(x)$ on the right has no absolute max.

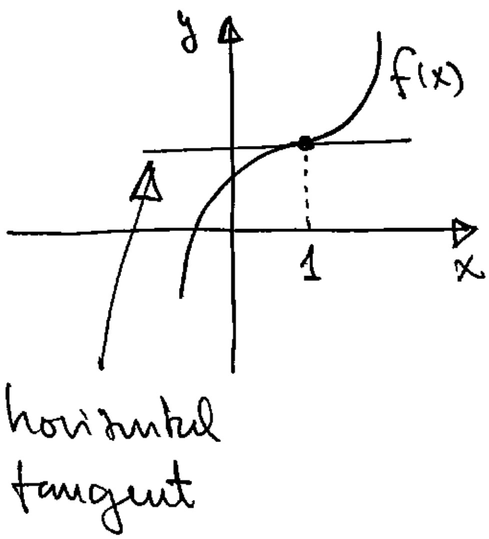


8. I is true, because differentiability at a implies continuity at a

II not true (f could have a horizontal tangent see question 1(d) or 9)

III true, since slope of tangent $= f'(2) = 0$

9.

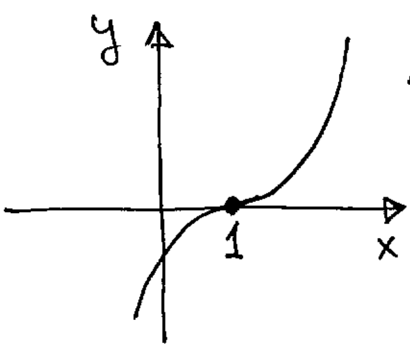


or:

$$y = (x-1)^3$$

$$y' = 3(x-1)^2$$

$$y'(1) = 0$$



but no extreme at $x=1$

10. $N'(t) = \dfrac{30000\,(100+t^2) - 30000t \cdot 2t}{(100+t^2)^2}$

$$= \frac{30000}{(100+t^2)^2}\,(100-t^2)$$

$N'(t)=0 \ldots \quad 100-t^2=0 \;\rightarrow\; t=\pm 10$

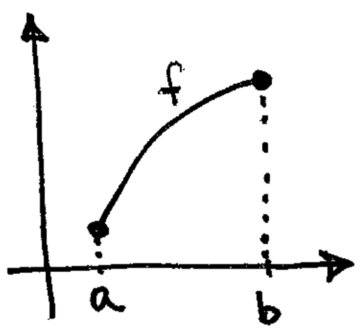
take $t=10$ only since assumption states $t \geq 0$

	9	10	11
N'	$+$		$-$
N	$\nearrow$		$\searrow$

$t=10$ is max

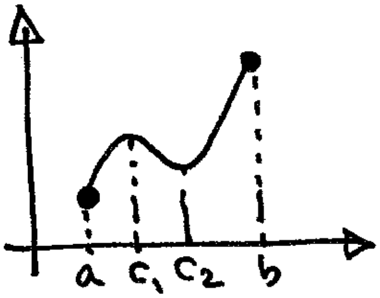
$N(10) = 5000 + \dfrac{30000 \cdot 10}{100+100} = 6500$ max population

11. (a)



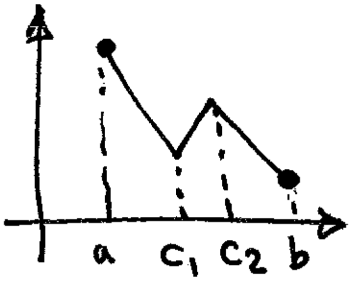
any increasing or decreasing function

(b)

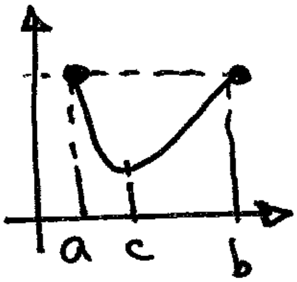


c_1, c_2 are critical points

or:

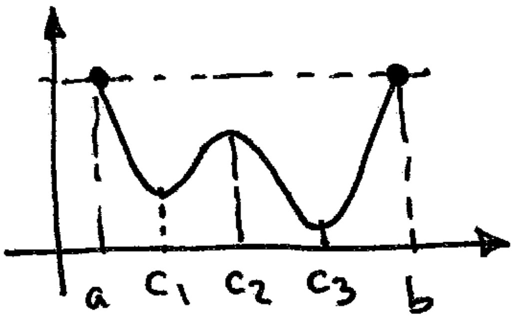


(c)



min. at c
max. at a and b

(d)



min. at c_3
c_1, c_2, c_3 are critical points