ASSIGNMENT 16.5

1 (a) $\lim_{x\to 0} \frac{\sin x}{\tan x} = \frac{0}{0}$ YES

(b) $\lim_{x\to 0} \frac{\cos x}{\tan x} = \frac{1}{0}$ NO

(c) $\lim_{x\to 0} x^x = 0^0$ YES

(d) $\lim_{x\to \infty} x^x = \infty^\infty$ NO

(e) $\lim_{x\to 1} x^x = 1^1$ NO

(f) $\lim_{x\to \infty} (x - \ln x) = \infty - \infty$ YES

(g) $\lim_{x\to \infty} (x + \ln x) = \infty + \infty$ NO

$= \frac{-\infty}{0}$ NOT INDETERMINATE!

2 (a) $\lim_{x\to 0^+} \frac{3\ln x}{x^2} = \lim_{x\to 0^+} (3\ln x)\cdot \frac{1}{x^2}$

$= 3(-\infty)\cdot \frac{1}{0^2} = 3(-\infty)(\infty) = -\infty$

(b) $\lim_{x\to 0} \frac{\arcsin x}{x} = \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{1/\sqrt{1-x^2}}{1} = 1$

(c) $\lim_{x\to\infty} \left(\sqrt{x^2-x} - x\right) = \infty - \infty \quad \frac{\cdot\ \sqrt{x^2-x} + x}{\cdot\ \sqrt{x^2-x} + x}$

$= \lim_{x\to\infty} \frac{\cancel{x^2} - x - \cancel{x^2}}{\sqrt{x^2-x} + x} = -\lim_{x\to\infty} \frac{x}{\sqrt{x^2-x} + x} \begin{matrix} \div x \\ \div x \end{matrix}$

$= -\lim_{x\to\infty} \frac{1}{\sqrt{\frac{x^2-x}{x^2}} + 1} = -\lim_{x\to\infty} \frac{1}{\sqrt{1-\underbrace{\cancel{\tfrac{1}{x}}}_{0}} + 1} = -\frac{1}{2}$

3 (a) $\lim_{x\to\infty} \frac{3x}{\ln(2+e^x)} = \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{3}{\frac{1}{2+e^x} \cdot e^x} = \lim_{x\to\infty} \frac{3(2+e^x)}{e^x}$

$= \lim_{x\to\infty} \left(\frac{6}{e^x} + \frac{3e^x}{e^x}\right) = \lim_{x\to\infty} \left(\frac{6}{e^x} + 3\right) = 3$

$\left(\text{since } \frac{6}{e^x} \to \frac{6}{e^\infty} = \frac{6}{\infty} = 0 \right)$

(b) $\lim_{x \to 0^+} (1-3x)^{1/x} = 1^{\infty}$

$$y = (1-3x)^{1/x}$$

$$\to \ln y = \frac{1}{x} \ln(1-3x)$$

$$\to \lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln(1-3x)}{x} = \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{\frac{1}{1-3x}(-3)}{1} = \lim_{x \to 0^+} \frac{-3}{1-3x} = -3$$

so $\lim_{x \to 0^+} (1-3x)^{1/x} = e^{-3}$

(c) $\lim_{x \to 0} \frac{e^{x^2}-1-x^2}{x^4} = \lim_{x \to 0} \frac{e^{x^2}\cdot 2x - 2x}{4x^3}$

$\frac{0}{0}$, indeterminate form; use LH

$$= \lim_{x \to 0} \frac{\cancel{2x}\,(e^{x^2}-1)}{{}_2\cancel{4}x^{\cancel{3}\,2}} = \lim_{x \to 0} \frac{e^{x^2}-1}{2x^2} = \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0} \frac{e^{x^2}\cdot 2\cancel{x}}{4\cancel{x}} = \lim_{x \to 0} \frac{e^{x^2}}{2} = \frac{1}{2}$$