ASSIGNMENT 16

1.

		at 0	at ∞
(a)	$3+2x$	3	$2x$
(b)	$7-2x^2$	7	$-2x^2$
(c)	$x-2e^x$	$-2e^x$	$-2e^x$
(d)	$3x-1000x^3+x^4$	$3x$	x^4
(e)	$x+2x^2+x^{-1}$	x^{-1}	$2x^2$
(f)	$x^{-1}+2x^{-3}+4x^{-5}$	$4x^{-5}$	x^{-1}

2. B approaches ∞ faster than A if

$$\lim_{x\to\infty} \frac{A}{B} = 0 \quad \underline{\underline{\text{or}}} \quad \lim_{x\to\infty} \frac{B}{A} = \infty$$

(a)
$$\lim_{x\to\infty} \frac{3+200x}{0.01x^2-100} = \lim_{x\to\infty} \frac{200x}{0.01x^2}$$

$$= \lim_{x\to\infty} \frac{200}{0.01x} = 0$$

$\rightarrow$ $0.01x^2-100$ is faster

(b) $\lim_{x\to\infty} \frac{25\ln x}{2\sqrt{x}} = \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{25}{x}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \frac{25}{\sqrt{x}} = \frac{25}{\infty} = 0$

$\rightarrow 2\sqrt{x}$ is faster

(c) $\lim_{x\to\infty} \frac{x+x^{10}}{e^x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{1+10x^9}{e^x} = \frac{\infty}{\infty} =$

$\overset{LH}{=} \lim_{x\to\infty} \frac{9\cdot 10x^8}{e^x} = \ldots$ apply LH again and again ... powers of x in numerator decrease

$\overset{LH}{=} \lim_{x\to\infty} \frac{\text{some constant}}{e^x} = 0$

$\rightarrow e^x$ is faster

(d) $\lim_{x\to\infty} \frac{\sqrt{x}}{x^{0.4}} = \lim_{x\to\infty} x^{0.1} = \infty^{0.1} = \infty$

switch:

$\lim_{x\to\infty} \frac{x^{0.4}}{\sqrt{x}} = \frac{1}{\infty} = 0 \longrightarrow \sqrt{x}$ is faster

3. B approaches 0 faster than A if $\lim \frac{B}{A} = 0$ ← also called: B is smaller than A

(a) $\lim_{x \to 0} \frac{0.01x^2}{200x} = \lim_{x \to 0} \frac{0.01x}{200} = 0$

$0.01x^2$ is faster

(b) $\lim_{x \to \infty} \frac{e^{-3x}}{x^{-2}} = \lim_{x \to \infty} \frac{x^2}{e^{3x}} = \frac{\infty}{\infty} \stackrel{LH}{=} \lim_{x \to \infty} \frac{2x}{3e^{3x}}$

$\stackrel{LH}{=} \lim_{x \to \infty} \frac{2}{9e^{3x}} = \frac{2}{9e^{\infty}} = \frac{2}{\infty} = 0$

→ e^{-3x} is faster

(c) $\lim_{x \to \infty} \frac{x^{-2}}{1000x^{-1}} = \frac{1}{1000} \lim_{x \to \infty} \frac{x}{x^2}$

$= \frac{1}{1000} \cdot \lim_{x \to \infty} \frac{1}{x} = 0$

→ x^{-2} is faster

(d) $\lim_{x \to 0} \frac{0.1x^3}{x^2} = 0$ → $0.1x^3$ is faster

4(a) $\lim_{x\to\infty} \dfrac{3x}{\ln(2+e^x)} = \dfrac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \dfrac{3}{\frac{1}{2+e^x}\cdot e^x} = \lim_{x\to\infty} \dfrac{\overset{6+3e^x}{3(2+e^x)}}{e^x}$

$= \dfrac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \dfrac{3e^x}{e^x} = \lim_{x\to\infty} 3 = 3$

(b) $\lim_{x\to 0} \dfrac{e^{x^2}-1-x^2}{x^4} = \dfrac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \dfrac{e^{x^2}(2x)-2x}{4x^3} = \lim_{x\to 0} \dfrac{2x(e^{x^2}-1)}{4x^3}$

$= \lim_{x\to 0} \dfrac{e^{x^2}-1}{2x^2} = \dfrac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \dfrac{e^{x^2}\cdot 2x}{4x} = \dfrac{1}{2} \lim_{x\to 0} e^{x^2} = \dfrac{1}{2}$

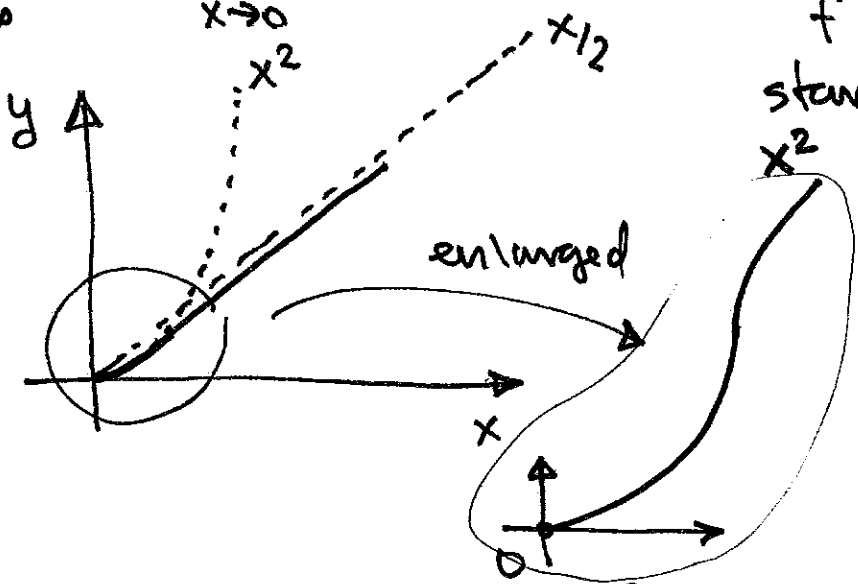
5.

(a) $f(x) = \dfrac{x^2}{2x+1}$ $\qquad f_0(x) = \dfrac{x^2}{1} = x^2$

$$f_\infty(x) = \frac{x^2}{2x} = \frac{x}{2}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} f_0(x) = 0$$

$$\lim_{x\to \infty} f(x) = \lim_{x\to 0} f_\infty(x) = \infty$$

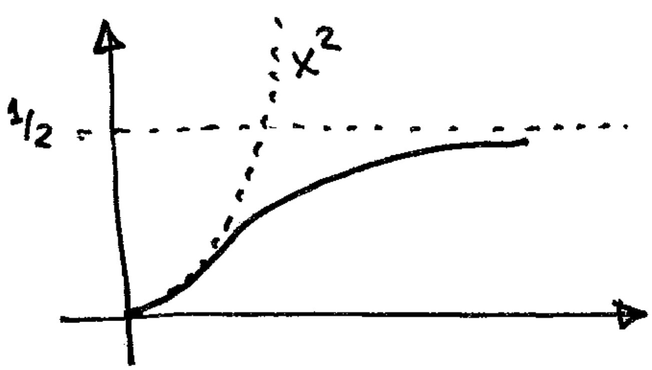


f: starts like x^2 then behaves like $\frac{x}{2}$

(b) $f(x) = \dfrac{x^2}{2x^2+1}$ $\qquad f_0(x) = \dfrac{x^2}{1} = x^2$

$$f_\infty(x) = \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\lim_{x\to 0} f(x) = 0 \qquad \lim_{x\to \infty} f(x) = \frac{1}{2}$$



starts like x^2
approaches $1/2$
as $x \to \infty$

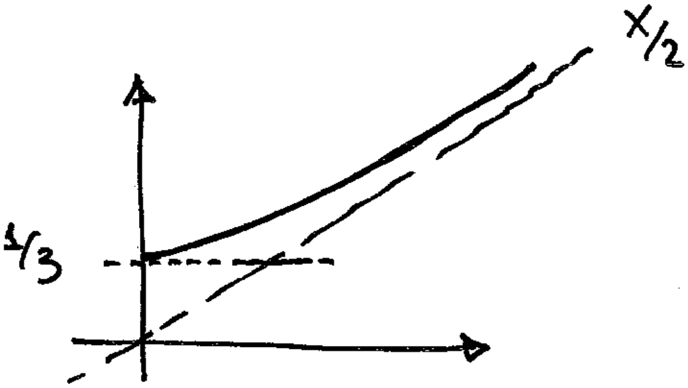
(c) $f(x) = \frac{1+x+x^2}{2x+3}$

$f_0(x) = \frac{1}{3}$

$f_\infty(x) = \frac{x^2}{2x} = \frac{x}{2}$

$\lim_{x \to 0} f(x) = \frac{1}{3}$

$\lim_{x \to \infty} f(x) = \infty$



(d) $f(x) = \frac{e^x}{2e^x + x}$

$f_0(x) = \frac{e^x}{2e^x} = \frac{1}{2}$

$f_\infty(x) = \frac{e^x}{2e^x} = \frac{1}{2}$

$\lim_{x \to 0} f(x) = \frac{1}{2}$

$\lim_{x \to \infty} f(x) = \frac{1}{2}$