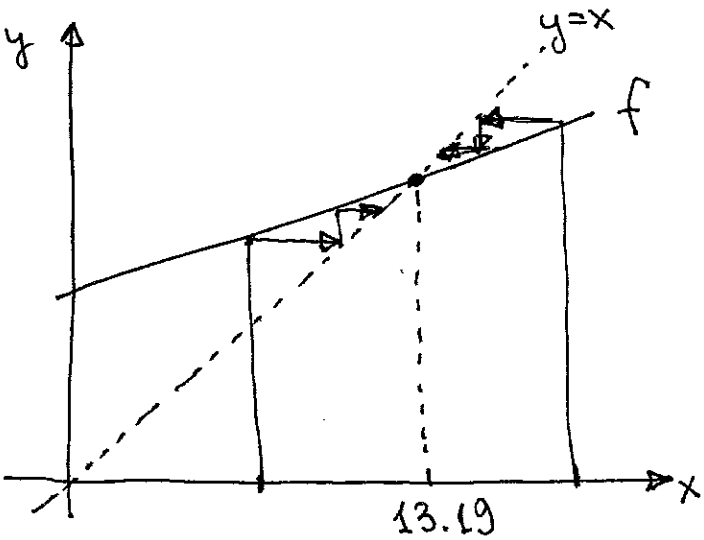
ASSIGNMENT 17

1. $f(x) = 0.53x + 6.2$

(a) equilibria: $0.53x + 6.2 = x$

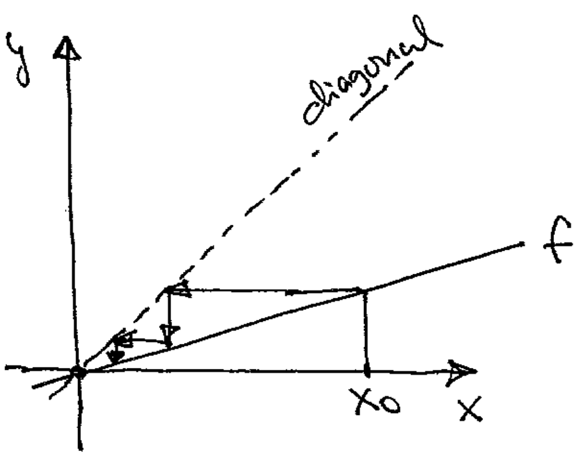
$0.47x = 6.2, \quad x = \frac{6.2}{0.47} \approx \underline{\underline{13.19}}$



$f'(x) = 0.53$ so $|f'(13.19)| < 1$ ... stable

cobwebbing: no matter where we start, iterations approach 13.19 $\rightarrow$ so stable

(b) $f(x) = 0.4x$ $\qquad f(x) = x \rightarrow 0.4x = x \rightarrow x = 0$ is the only equilibrium



$f'(x) = 0.4$ so $|f'(x)| < 1$ ... stable

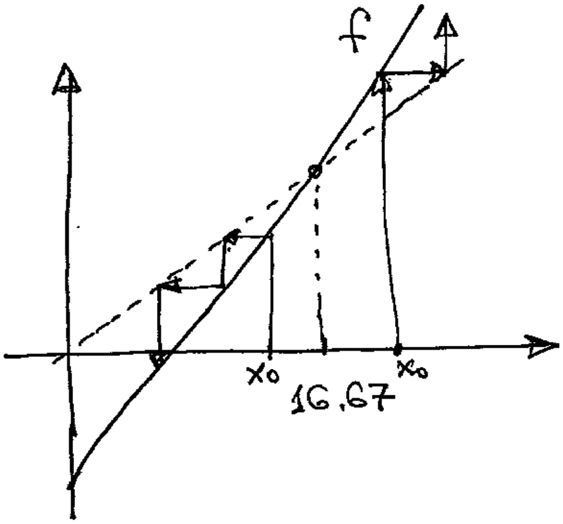
cobwebbing ... take positive initial condition x_0 $\rightarrow$ moves towards 0 $\rightarrow$ stable

(c) $f(x) = 1.3x - 5$ $\qquad f(x) = x \rightarrow 1.3x - 5 = x$

$$0.3x = 5$$

$$x = 5/0.3 \approx 16.67 \text{ equilibrium}$$

$$f'(x) = 1.3 \rightarrow |f'(x)| > 1, \text{ so unstable}$$

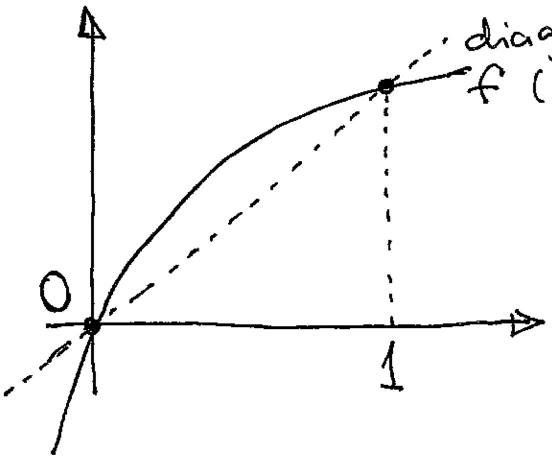


cobwebbing: x_0 near 16.67
iterations move away
→ unstable

(d) $f(x) = \dfrac{2x}{x+1}$ $\qquad f(x) = x \rightarrow \dfrac{2x}{x+1} = x$

so $x\left(\dfrac{2}{x+1} - 1\right) = 0 \rightarrow x = 0$, $x = 1$

two equilibria



diagonal

f (obtained by plotting points)

has to intersect the diagonal at $x=0$ and $x=1$
so goes through (0,0) and (1,1); then plot points,

x	$1/2$	$1/4$	...
$y = \frac{2x}{x+1}$	$2/3$	$2/5$	...

$$f'(x) = \frac{2(x+1) - 2x \cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

(so f is increasing)

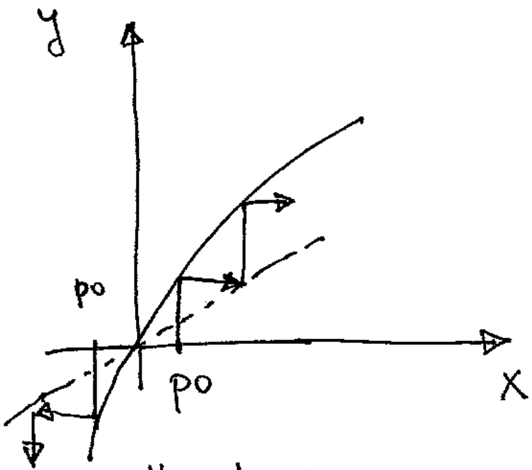
alternative to plotting points:

$$f''(x) = 2(-2)(x+1)^{-3} = \frac{-4}{(x+1)^3} < 0 \quad \text{for } x > 0$$

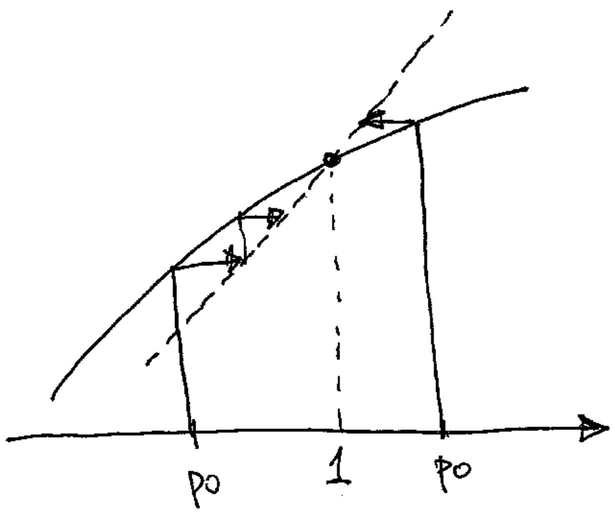
ie f is concave down!

stability ... $0 \to f'(0) = 2$, $|f'(0)| > 1$ so unstable
$1 \to f'(1) = 1/2$, $|f'(1)| < 1$ so stable

cobwebbing:

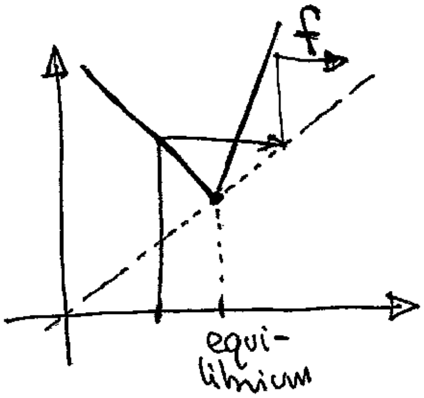


iterations move away from po
$\Rightarrow$ 0 is unstable



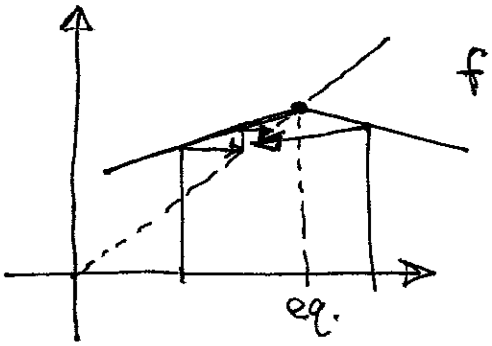
iterations move towards 1
$p = 1$ is stable

2. (a)



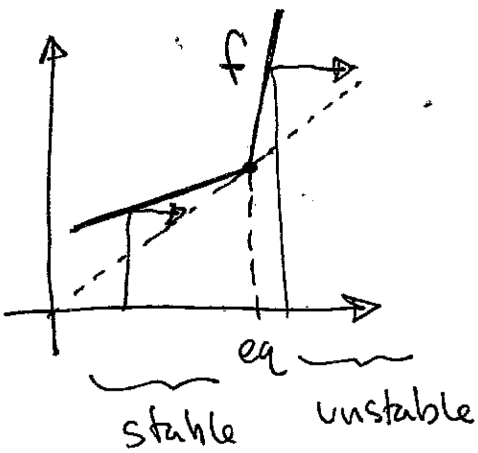
f can be like absolute value graph with corner sitting on the diagonal

(b)



stable
cobwebbing to the right of equilibrium moves to the left of it, and then approaches eq.

(c)



combine ideas in (a) and (b)

3. (a) $f(x) = 0.4x + 1.2$ $\quad f(x) = x \to 0.4x + 1.2 = x$

so $0.6x = 1.2$, $x = 2$

$f'(x) = 0.4 \to |f'(2)| = 0.4 < 1$, so $x = 2$ is stable inverse: $x_{t+1} = 0.4x_t + 1.2$

so $x_t = \dfrac{x_{t+1} - 1.2}{0.4} = 2.5x_{t+1} - 3$

ie $f^{-1}(x) = 2.5x - 3$

$(f^{-1}(x))' = 2.5$, since > 1 $x=2$ is unstable for inverse system

(b) $f(x) = \dfrac{2x}{1+0.01x} = x \rightarrow x\left(\dfrac{2}{1+0.01x} - 1\right) = 0$

so $\underline{\underline{x=0}}$ or $1+0.01x = 2$, so $\underline{\underline{x=100}}$

$$f'(x) = \frac{2(1+0.01x) - 2x \cdot 0.01}{(1+0.01x)^2} = \frac{2}{(1+0.01x)^2}$$

$x=0 \ldots \ |f'(0)| = 2 > 1 \ldots$ unstable

$x=100 \ldots |f'(100)| = 1/2 < 1 \ldots$ stable

inverse: $p_{t+1} = \dfrac{2p_t}{1+0.01p_t} \rightarrow p_{t+1} + 0.01p_t p_{t+1} = 2p_t$

$\Rightarrow p_{t+1} = p_t(2 - 0.01p_{t+1})$ ie

$p_t = \dfrac{p_{t+1}}{2-0.01p_{t+1}}$ so $f^{-1}(x) = \dfrac{x}{2-0.01x}$

$$(f^{-1})'(x) = \frac{2-0.01x - x(-0.01)}{(2-0.01x)^2} = \frac{2}{(2-0.01x)^2}$$

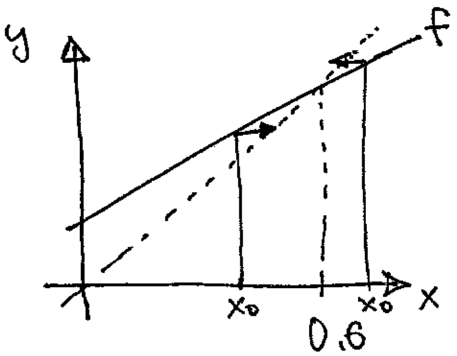
$|(f^{-1})'(0)| = \frac{2}{4} = \frac{1}{2} < 1 \ldots$ stable

$|(f^{-1})'(100)| = \frac{2}{1} = 2 > 1 \ldots$ unstable

4. (a) $f(x) = 1 + 0.8(x - 1.1) = 0.8x + 0.12$

$f(x) = x \to 0.8x + 0.12 = x \to x = \frac{0.12}{0.2} = 0.6$

$f'(x) = 0.8 \to |f'(0.6)| < 1$ ... stable

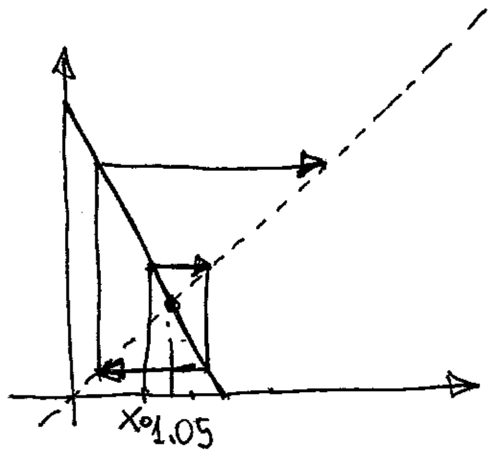


iterative move towards 0.6 $\to$ stable

(b) $f(x) = 1 - 1.2(x - 1.1) = -1.2x + 2.32$

equilibrium: $-1.2x + 2.32 = x \to x = \frac{2.32}{2.2} \approx 1.05$

$|f'(x)| = |-1.2| = 1.2 > 1$ ... unstable

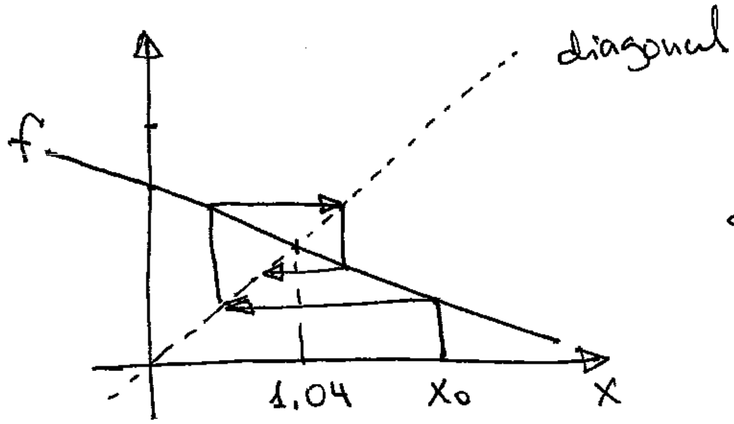


unstable

(c) $f(x) = 1 - 0.8(x - 1.1) = -0.8x + 1.88$

$f(x) = x \rightarrow -0.8x + 1.88 = x \rightarrow x = \frac{1.88}{1.8} = 1.04$

$|f'(1.04)| = |-0.8| = 0.8 < 1$ stable

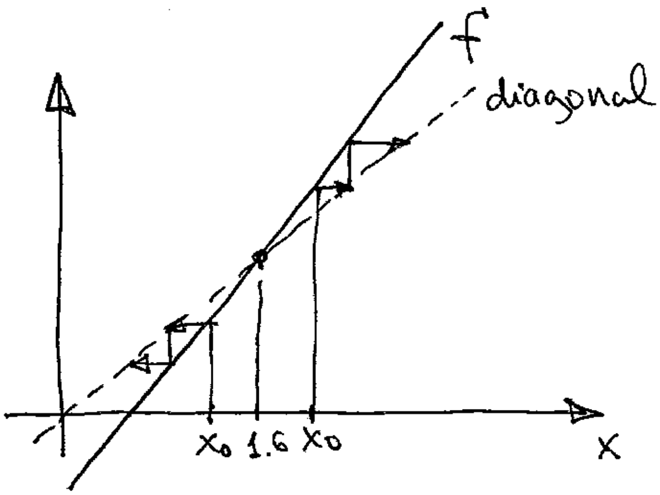


stable

(d) $f(x) = 1 + 1.2(x - 1.1) = 1.2x - 0.32$

$f(x) = x \rightarrow 1.2x - 0.32 = x$, $x = \frac{-0.32}{-0.2} = 1.6$

$|f'(1.6)| = |1.2| = 1.2 > 1$ unstable



unstable

END