ASSIGNMENT 18

1.(a)

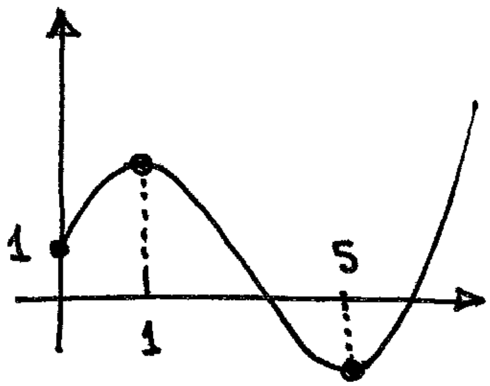
	$(-\infty,1)$	$(1,5)$	$(5,\infty)$
f'	$+$	$-$	$+$
f	$\nearrow$	$\searrow$	$\nearrow$

incr. on $(-\infty,1)$
$(5,\infty)$

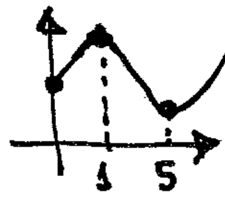
decr. on $(1,5)$

(b) max. at $x=1$

(c)



many possibilities could be



etc.

2.(a) $f(x) = x^{-1/2} + x^{-1} + x^{-3/2}$

so $\displaystyle\int f(x)\,dx = \frac{x^{1/2}}{1/2} + \ln|x| + \frac{x^{-1/2}}{-1/2} + C$

$\displaystyle= 2\sqrt{x} + \ln|x| - \frac{2}{\sqrt{x}} + C$

(b) $\displaystyle\int y\,dx = \frac{\pi^x}{\ln\pi} + \frac{x^{\pi+1}}{\pi+1} + \pi^\pi x + C$

(c) $\displaystyle\int f(x)\,dx = -3\cos x + 4\sin x + 5x + C$

(d) $\int f(x)\,dx = e^x + \frac{x^{-1}}{-1} + C = e^x - \frac{1}{x} + C$

(e) $\int y\,dx = \sec x - x + C$

(f) $f(x) = \frac{x - 2x^2 + 1}{x^{1/2}} = x^{1/2} - 2x^{3/2} + x^{-1/2}$

so $\int f(x)\,dx = \frac{x^{3/2}}{3/2} - 2\,\frac{x^{5/2}}{5/2} + \frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3}x^{3/2} - \frac{4}{5}x^{5/2} + 2x^{1/2} + C$

3. (a) $= \tan x + 3\sin x + C$

(b) $= \int\left(\frac{1.4}{5}x^{-1/2} + 2x^{-1/5}\right)dx$

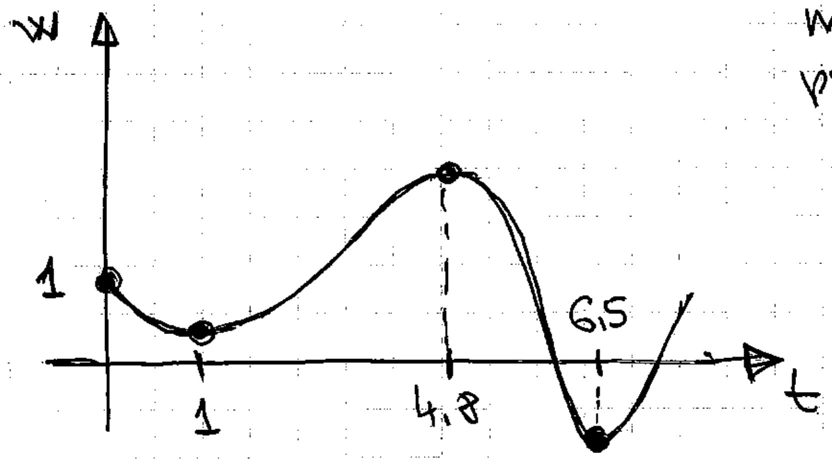
$= 0.28 \cdot \frac{x^{1/2}}{1/2} + 2 \cdot \frac{x^{4/5}}{4/5} + C = 0.56\sqrt{x} + \frac{5}{2}x^{4/5} + C$

(c) $= 6\arctan x + C$

4.

approximation from a given graph

		1		4.8		6.5	
$\frac{dw}{dt}$	$-$		$+$		$-$		$+$
w	↘		↗		↘		↗
		min		max		min	



many possibilities!!

5. (a) $y' = \frac{1}{2}(e^x - e^{-x})$, $y'' = \frac{1}{2}(e^x + e^{-x})$

check $y'' \stackrel{?}{=} \sqrt{1+(y')^2}$

$$1+(y')^2 = 1 + \frac{1}{4}(e^x - e^{-x})^2 = 1 + \frac{1}{4}(e^{2x} - 2 + e^{-2x})$$

$$= \frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}$$

$$= \frac{1}{4}(e^x + e^{-x})^2$$

so $\sqrt{1+(y')^2} = \frac{1}{2}(e^x + e^{-x}) = \underline{\underline{y''}}$

for $y = \frac{1}{2}(e^x - e^{-x})$:

$$y' = \frac{1}{2}(e^x + e^{-x}), \quad y'' = \frac{1}{2}(e^x - e^{-x})$$

as above: $1+(y')^2 = 1 + \frac{1}{2}(e^x + e^{-x})^2$

$$= 1 + \frac{1}{4}(e^{2x} + 2 + e^{-2x})$$

$$= \frac{1}{4}e^{2x} + \frac{3}{2} + \frac{1}{4}e^{-2x}$$

= does not simplify as above, ie is not the square of something

check $\quad y'' \stackrel{?}{=} \sqrt{1+(y')^2}$

$$\frac{1}{2}\left(e^{x}-e^{-x}\right) \stackrel{?}{=} \sqrt{\frac{1}{4}e^{2x}+\frac{3}{2}+\frac{1}{4}e^{-2x}}$$

take $x=0$:

$$0 \stackrel{?}{=} \sqrt{\frac{3}{2}+\frac{1}{4}+\frac{1}{4}} = \sqrt{2}$$

not true, so answer is <u>NO</u>

(b) $\quad f'(t) = \dfrac{4}{1+t}$ by the chain rule.

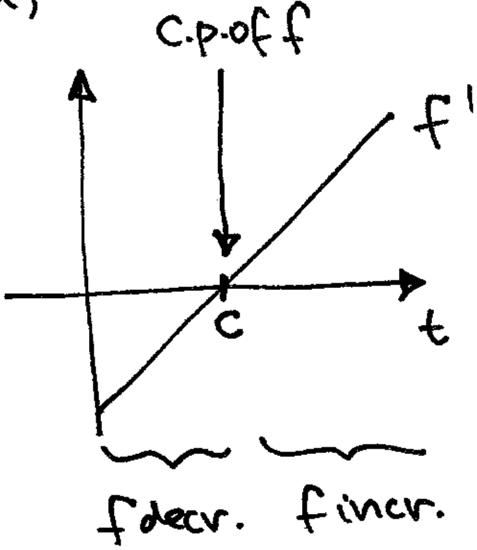
answer: <u>YES</u>

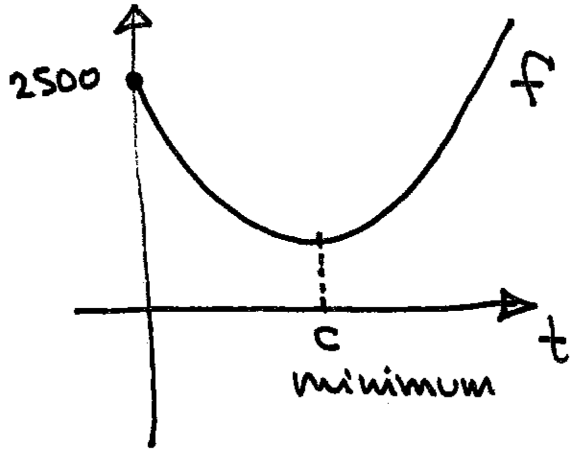
6. (a) $\quad T'(t) = k \cdot \dfrac{1}{T(t)^2}, \quad T(0)=0$

(b) $\quad S'(t) = k\,S(t)\left(15{,}000 - S(t)\right), \quad S(0)=1$

(c) $\quad T'(t) = k\left(300 - T(t)\right), \quad T(0) = 20^{\circ}C$

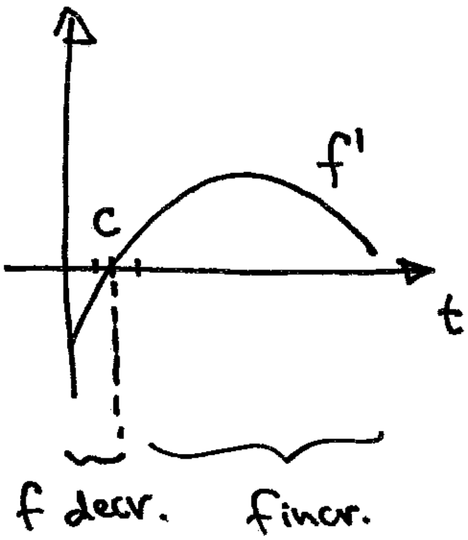
7. (a)

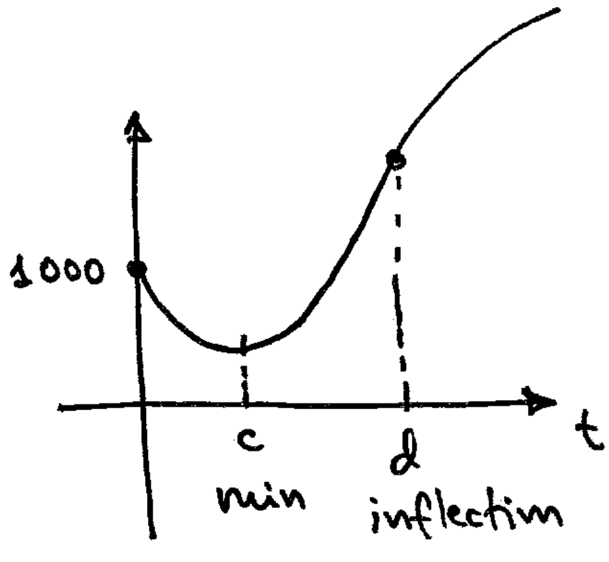


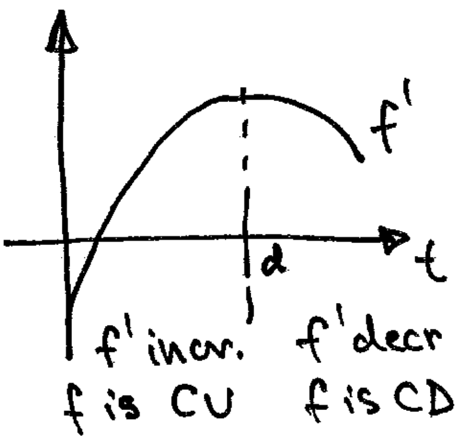


(antiderivative of a line is a parabola)

(b)







8. (a) If $f(t)$ is unknown function:

$f'(t) =$ function of t (measured rate of change) is pure-time

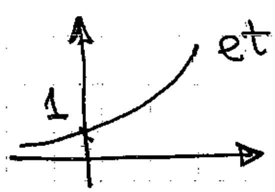
$f'(t) =$ function of f, no explicit appearance of t is autonomous

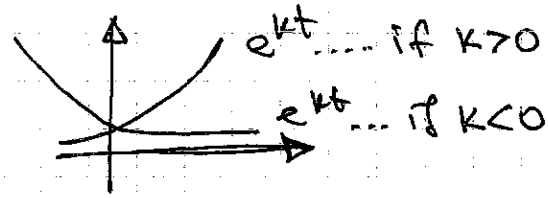
(b) $f'(t) = \text{constant}$

(c) $f'(t) = t^2 \cdot f(t)$ ← not pure-time

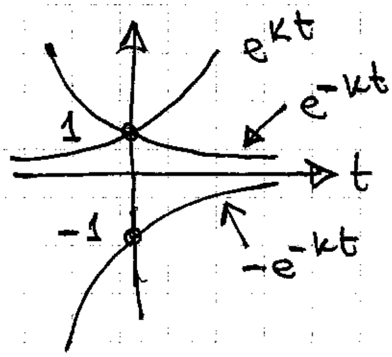
↑ not autonomous

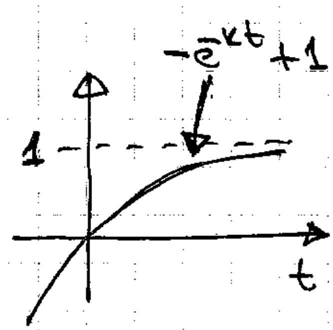
(d) $P(t) = M - Me^{-kt} = M(1 - e^{-kt})$

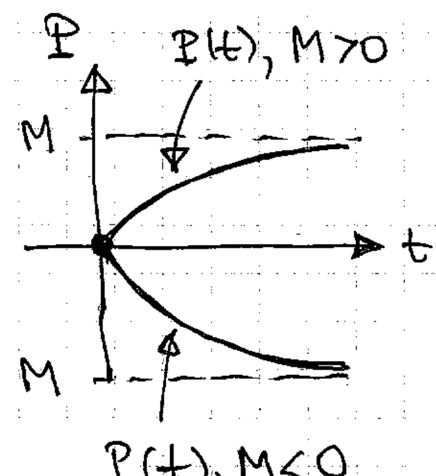


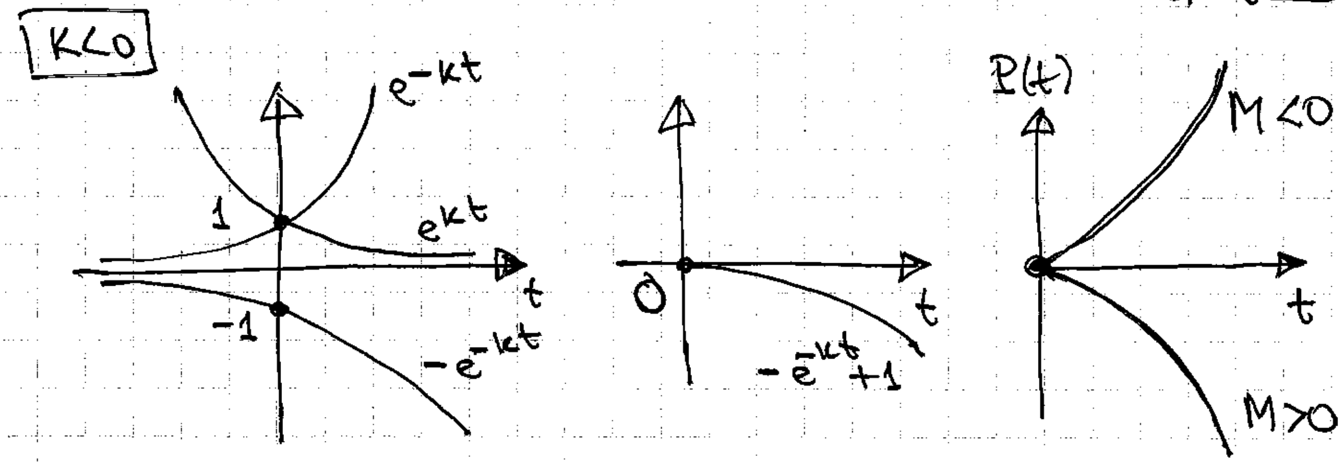


$\boxed{k>0 \text{ case}}$









9. $P'(t) = 0.02\, P(t)$

guess: $P(t) = C \cdot e^{0.02t}$

check: $P'(t) = C \cdot e^{0.02t}\,(0.02)$

$0.02\, P(t) = 0.02 \cdot C\, e^{0.02t}$ equal

so $P(t) = C\, e^{0.02t}$

$P(0) = 240 \longrightarrow 240 = C \cdot e^{0}, \quad C = 240$

so $\underline{\underline{P(t) = 240\, e^{0.02t}}}$