ASSIGNMENT 19

1.(a) $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$

we write $\int f(x)dx = F(x) + C$

(b) $\left(\frac{1}{4}(1-2x+2x^2)e^{2x}\right)'$

$= \frac{1}{4}\left((-2+4x)e^{2x} + (1-2x+2x^2)e^{2x}\cdot 2\right)$

$= \frac{1}{4}e^{2x}\left(\cancel{-2} + \cancel{4x} + \cancel{2} - \cancel{4x} + 4x^2\right) = x^2 e^{2x}$

(c) No. The derivative $\left(\frac{1}{1+x^2} + C\right)' = (-1)(1+x^2)^{-2}\cdot 2x$

$= \frac{-2x}{(1+x^2)^2}$ is not equal to $\arctan x$.

note:
It IS TRUE that $\int \frac{1}{1+x^2}dx = \arctan x + C$

2. $\frac{dA}{dt} = 24.6t^3 \rightarrow A(t) = \int 24.6t^3 = 24.6\cdot\frac{t^4}{4} + C$

So $A(t) = 6.15t^4 + C$

$A(0) = 10 \rightarrow 10 = 6.15(0)^4 + C$, So $C = 10$

thus $\boxed{A(t) = 6.15t^4 + 10}$

3. $f'(x) = \frac{4}{x} \rightarrow f(x) = \int \frac{4}{x}dx = 4\ln|x| + C$

it is given that $x < 0 \Rightarrow |x| = -x$ and so

$f(x) = 4\ln(-x) + C$

$f(-2) = 4 \rightarrow 4\ln 2 + C = 4 \Rightarrow C = 4 - 4\ln 2$

so $\boxed{f(x) = 4\ln(-x) + 4 - 4\ln 2}$

4. $f'(x) = 6^x - 4 \rightarrow f(x) = \int (6^x - 4)\,dx$

so $f(x) = \dfrac{6^x}{\ln 6} - 4x + C$

$f(3) = 12 \longrightarrow \dfrac{6^3}{\ln 6} - 4(3) + C = 12$

$C = 24 - \dfrac{6^3}{\ln 6} \approx -96.55$

so $f(x) = \dfrac{6^x}{\ln 6} - 4x - 96.55$

5. (a) $\int \frac{1}{4}\,dx = \frac{1}{4}x + C$

(b) $\int \sqrt{7}\,dx = \sqrt{7}\,x + C$

(c) $\int (2^x + x^2)\,dx = \dfrac{2^x}{\ln 2} + \dfrac{x^3}{3} + C$

6. (a) $\int \left(2\cos\left(\frac{x}{3}\right) + 4\cos(3x)\right) dx$

$= 2 \cdot \dfrac{\sin\left(\frac{x}{3}\right)}{\frac{1}{3}} + 4 \cdot \dfrac{\sin 3x}{3} + C$

$= 6\sin(x/3) + \frac{4}{3}\sin 3x + C$

(b) $= e^x + \frac{1}{2}e^{2x} + \dfrac{1}{\frac{1}{2}}e^{x/2} + C = e^x + \frac{1}{2}e^{2x} + 2e^{x/2} + C$

(c) $= \dfrac{\ln|1+7x|}{7} + C$

(d) $= \dfrac{(1+7x)^5}{5} \cdot \dfrac{1}{7} + C$