

Math 1LS3**Assignment 20.5**

Practice with integration: 6.2, 6.3, 6.4

1. Find the following integrals.

$$(a) \int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$$

$$(b) \int_1^2 \frac{12}{x^2} dx = 12 \int_1^2 x^{-2} dx = 12 \left. \frac{x^{-1}}{-1} \right|_1^2 = -12 \left. \frac{1}{x} \right|_1^2 \\ = -12 \left(\frac{1}{2} - 1 \right) = 6$$

$$(c) \int \left(3 - \frac{7}{\sqrt[4]{x}} \right) dx = \int 3 dx - 7 \int x^{-1/4} dx = 3x - 7 \cdot \frac{x^{3/4}}{3/4} + C \\ = 3x - \frac{28}{3} x^{3/4} + C$$

$$(d) \int_0^4 (7 - 2x)^2 dx = \int_0^4 (49 - 28x + 4x^2) dx = 49x - 14x^2 + 4 \frac{x^3}{3} \Big|_0^4 \\ = 49(4) - 14(4^2) + 4 \frac{(4^3)}{3} = \frac{172}{3}$$

$$(e) \int \frac{x^3 - 3x + 1}{x} dx = \int \left(x^2 - 3 + \frac{1}{x} \right) dx = \frac{x^3}{3} - 3x + \ln|x| + C$$

$$(f) \int \frac{(x^2 - 3)^2}{7x^3} dx = \frac{1}{7} \int \frac{x^4 - 6x^2 + 9}{x^3} dx = \frac{1}{7} \int \left(x - \frac{6}{x} + \frac{9}{x^3} \right) dx \\ = \frac{1}{7} \left(\frac{x^2}{2} - 6 \ln|x| + 9 \frac{x^{-2}}{-2} \right) = \frac{1}{7} \left(\frac{x^2}{2} - 6 \ln|x| - \frac{9}{2x^2} \right) + C$$

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$$(g) \int \frac{1}{23} \frac{dx}{x} = \frac{1}{23} x + C$$

$$(h) \int \frac{1}{23} \frac{dt}{t} = \frac{1}{23} t + C$$

$$(i) \int \frac{1}{23} \frac{du}{u} = \frac{1}{23} u + C$$

NOTE: dt, du, dx tell us
what the variable is

$$(j) \int_1^{11} \ln 4 \, dt = (\ln 4) t \Big|_1^{11} = (\ln 4)(11-1) = 10 \ln 4$$

$$(k) \int (e^x - 2^x + 6^x) \, dx = e^x - \frac{2^x}{\ln 2} + \frac{6^x}{\ln 6} + C$$

$$(l) \int_0^1 4e^x \, dx = 4e^x \Big|_0^1 = 4(e^1 - e^0) = 4(e-1)$$

$$(m) \int_0^1 4e^t \, dt = 4e^t \Big|_0^1 = 4(e-1)$$

$$(n) \int_0^1 4e^u \, du = 4e^u \Big|_0^1 = 4(e-1)$$

NOTE:

definite
integral is a
real number

so does not matter
what variable is
used

$$(o) \int (4e^2 - 24x) \, dx = 4e^2 x - 12x^2 + C$$

constant

$$(p) \int \frac{14^x - 3}{7} \, dx = \frac{1}{7} \int (14^x - 3) \, dx = \frac{1}{7} \left(\frac{14^x}{\ln 14} - 3x \right) + C$$

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$$(q) \int \cos x \, dx = \sin x + C$$

$$(r) \int_0^1 \cos 3x \, dx = \frac{1}{3} \sin 3x \Big|_0^1 = \frac{1}{3} \sin 3 - \frac{1}{3} \sin 0 = \frac{1}{3} \sin 3$$

radians
↓
0

$$(s) \int \cos(-4x) \, dx = -\frac{1}{4} \sin(-4x) + C$$

$$(t) \int \sec^2 x \, dx = \tan x + C$$

$$(u) \int_0^{1/3} \sec^2 3x \, dx = \frac{1}{3} \tan 3x \Big|_0^{1/3} = \frac{1}{3} \tan 1 - \frac{1}{3} \tan 0 = \frac{1}{3} \tan 1$$

0

$$(v) \int \sec^2(-4x) \, dx = -\frac{1}{4} \tan(-4x) + C$$

$$(w) \int \sec x \tan x \, dx = \sec x + C$$

$$(x) \int_1^4 \sec 4x \tan 4x \, dx = \frac{1}{4} \sec 4x$$

$$(y) \int (\sin 2x - \cos 3x + 4) \, dx = -\frac{1}{2} \cos 2x - \frac{1}{3} \sin 3x + 4x + C$$

$$(z) \int_0^1 (10 \sin(\pi x) - 2) \, dx = \left(10 \cdot \frac{-1}{\pi} \cos(\pi x) - 2x \right) \Big|_0^1$$

$$= \left(-\frac{10}{\pi} (-1) - 2 \right) - \left(-\frac{10}{\pi} - 0 \right) = \frac{20}{\pi} - 2$$

$$(aa) \int_0^1 \frac{1}{1+x^2} dx = \arctan x \Big|_0^1 = \underbrace{\arctan 1}_{\pi/4} - \underbrace{\arctan 0}_0 = \frac{\pi}{4}$$

$$(ab) \int \left(2 - \frac{4}{1+x^2} \right) dx = 2x - 4 \arctan x + C$$

$$(ac) \int \frac{1}{1+4x^2} dx = \int \frac{1}{1+(2x)^2} dx = \frac{1}{2} \arctan 2x + C$$

$$(ad) \int \frac{5}{1+6x^2} dx = 5 \cdot \int \frac{1}{1+(\sqrt{6}x)^2} dx = 5 \cdot \frac{1}{\sqrt{6}} \arctan(\sqrt{6}x) + C$$

$$(ae) \int 45(1-x^2)^{-1/2} dx = 45 \int \frac{1}{\sqrt{1-x^2}} dx = 45 \arcsin x + C$$

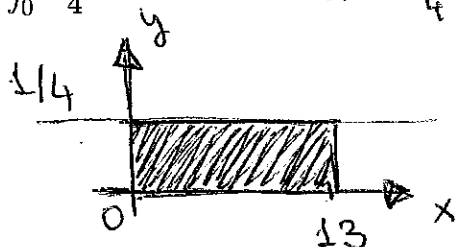
$$(af) \int \left(\frac{3}{\sqrt{1-x^2}} + \sin x \right) dx = 3 \arcsin x - \cos x + C$$

$$(ag) \int \frac{1}{\sqrt{1-9x^2}} dx = \int \frac{1}{\sqrt{1-(3x)^2}} dx = \frac{1}{3} \arcsin 3x + C$$

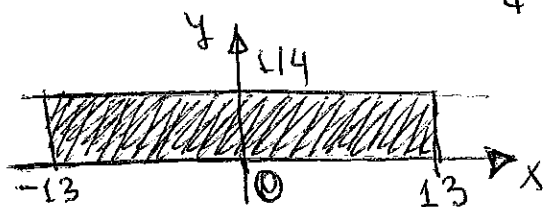
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2. Evaluate each integral by interpreting it as area or as a difference of areas.

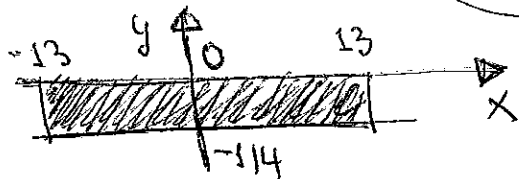
(a) $\int_0^{13} \frac{1}{4} dx = \text{area} = \frac{1}{4} \cdot 13 = 13/4$



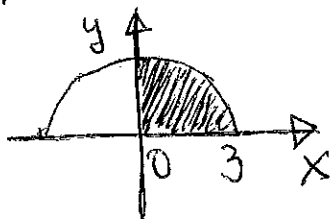
(b) $\int_{-13}^{13} \frac{1}{4} dx = \text{area} = 26 \cdot \frac{1}{4} = 13/2$



(c) $\int_{-13}^{13} \left(-\frac{1}{4}\right) dx \rightarrow -\text{area} = -26 \cdot \frac{1}{4} = -13/2$

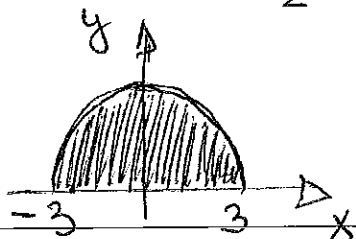


(d) $\int_0^3 \sqrt{9-x^2} dx \rightarrow y = \sqrt{9-x^2} \rightarrow x^2 + y^2 = 9$



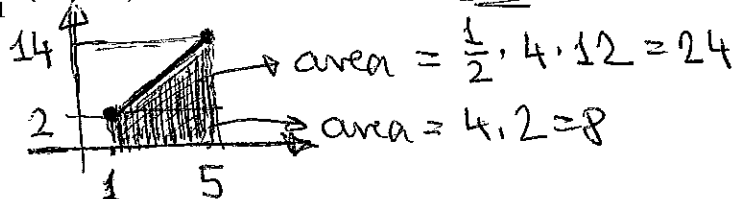
$$\int_0^3 \sqrt{9-x^2} dx = \frac{1}{4} \pi (3)^2 = \frac{9\pi}{4}$$

(e) $\int_{-3}^3 \sqrt{9-x^2} dx = \frac{1}{2} \pi (3)^2 = \frac{9\pi}{2}$

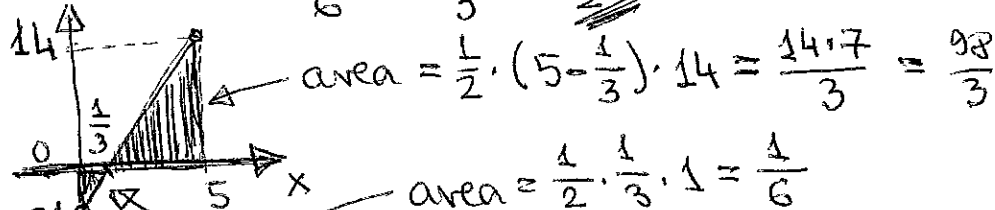


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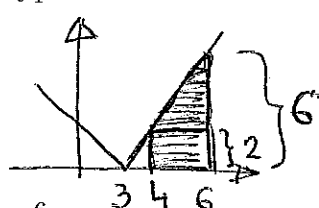
(f) $\int_1^5 (3x-1) dx = 8 + 24 = \underline{32}$



(g) $\int_0^5 (3x-1) dx = -\frac{1}{6} + \frac{98}{3} = \underline{\underline{\frac{65}{2}}}$

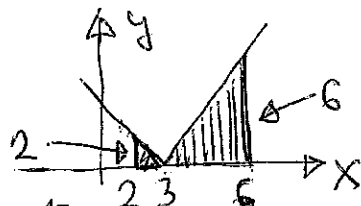


(h) $\int_4^6 |2x-6| dx$

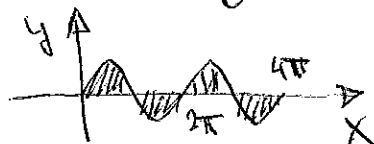


$\int_4^6 |2x-6| = \text{area of rectangle} + \text{area of triangle} = 4 + 4 = 8$

(i) $\int_2^6 |2x-6| dx = \text{sum of areas} = \frac{1}{2} \cdot 1 \cdot 2 + \frac{1}{2} \cdot 3 \cdot 6 = 10$

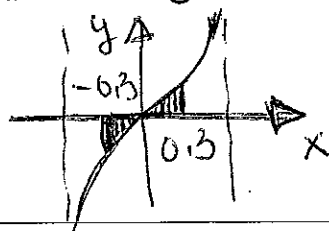


(j) $\int_0^{4\pi} \sin x dx = 0$



difference of areas

(k) $\int_{-0.3}^{0.3} \tan x dx = 0$



difference of areas

THE END