

ASSIGNMENT 21

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$$1.(a) \int_0^{\pi} (\sin x - 2 \cos x) dx = (-\cos x - 2 \sin x) \Big|_0^{\pi} = 1 - (-1) = 2$$

$$(b) \int_1^9 x^{-1/2} dx = 2x^{1/2} \Big|_1^9 = 2\sqrt{9} - 2\sqrt{1} = 4$$

$$(c) \int_0^1 \frac{6}{1+x^2} dx = 6 \arctan x \Big|_0^1 = 6 \cdot \frac{\pi}{4} - 0 = \frac{3\pi}{2}$$

$$(d) \int_0^1 11e^x dx = 11e^x \Big|_0^1 = 11e - 11$$

$$(e) \int_{-2}^{-1} x^{-1} dx = \ln|x| \Big|_{-2}^{-1} = \ln 1 - \ln 2 = -\ln 2$$

$$(f) \int_1^2 \frac{x^2+1}{x} dx = \int_1^2 \left(x + \frac{1}{x}\right) dx = \left(\frac{x^2}{2} + \ln|x|\right) \Big|_1^2$$

$$= \left(2 + \ln 2\right) - \left(\frac{1}{2} + \ln 1\right) = \frac{3}{2} + \ln 2$$

$$2.(a) = \left\{ \begin{array}{l} u = x^2 + x + 2 \\ du = (2x+1)dx \end{array} \right\} = \int \frac{du}{u^2} = -\frac{1}{u} + C$$

$$= -\frac{1}{x^2+x+2} + C$$

$$(b) = \left\{ \begin{array}{l} u = 3 - e^x \\ du = -e^x dx \end{array} \right\} = \int \frac{-du}{u} = -\ln|u| + C = \underline{\underline{-\ln|3 - e^x| + C}}$$

$$(c) = \left\{ \begin{array}{l} u = 1 + \sqrt{x} \\ du = \frac{1}{2\sqrt{x}} dx \rightarrow dx = 2\sqrt{x} du \end{array} \right\} = \int \frac{u^2}{\cancel{\sqrt{x}}} 2\sqrt{x} du$$

$$= 2 \int u^2 du = \frac{2u^3}{3} + C = \underline{\underline{\frac{2}{3}(1 + \sqrt{x})^3 + C}}$$

$$(d) = \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right\} = \int_0^{\ln 2} \frac{u^4}{1} du = \frac{u^5}{5} \Big|_0^{\ln 2} = \underline{\underline{\frac{(\ln 2)^5}{5}}}$$

$$\text{when } x=1 \rightarrow u = \ln 1 = 0$$

$$\text{when } x=2 \rightarrow u = \ln 2$$

$$3.(a) \quad \int_0^1 \frac{1}{1+x^2} dx = \arctan x \Big|_0^1$$

$$= \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

$$(b) \quad \int_0^1 \frac{x}{1+x^2} dx = \left\{ \begin{array}{l} u = 1+x^2 \\ du = 2x dx \end{array} \right\}$$

$$= \int_1^2 \frac{x}{u} \frac{du}{2x} = \frac{1}{2} \int_1^2 \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| \Big|_1^2 = \frac{1}{2} \ln 2 - \frac{1}{2} \ln 1 = \frac{1}{2} \ln 2$$

$$(c) \quad \int_0^1 \frac{x^2}{1+x^2} dx = \quad x^2 + 1 \frac{\sqrt{x^2}}{x^2 + 1} \frac{1}{-1}$$

$$= \int_0^1 \left(1 - \frac{1}{1+x^2} \right) dx$$

$$= (x - \arctan x) \Big|_0^1$$

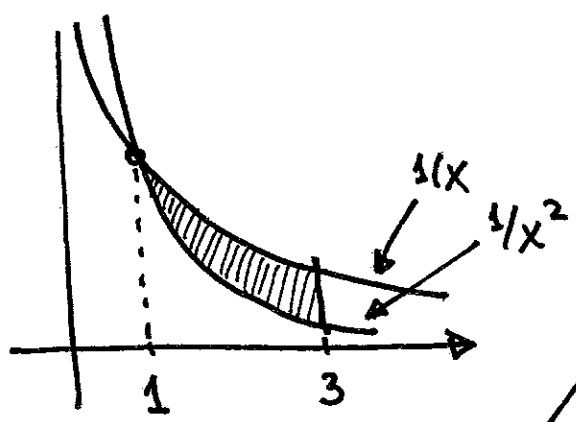
$$= (1 - \arctan 1) - (0 - \underbrace{\arctan 0}_0) = 1 - \frac{\pi}{4}$$

4. $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right\}$

$$= \int_{\ln e}^{\ln e^4} \frac{1}{\sqrt{u}} du = \int_1^4 u^{-1/2} du = 2\sqrt{u} \Big|_1^4$$

$$= 2\sqrt{4} - 2\sqrt{1} = 2$$

5.



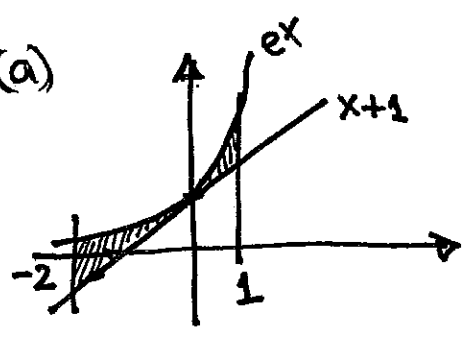
$$A = \int_1^3 \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

~~$A = \left(\ln|x| - \frac{x^{-2}}{-2} \right) \Big|_1^3 = \left(\ln|x| + \frac{1}{2x^2} \right) \Big|_1^3$~~

$$A = \left(\ln|x| - \frac{x^{-1}}{-1} \right) \Big|_1^3 = \left(\ln|x| + \frac{1}{x} \right) \Big|_1^3$$

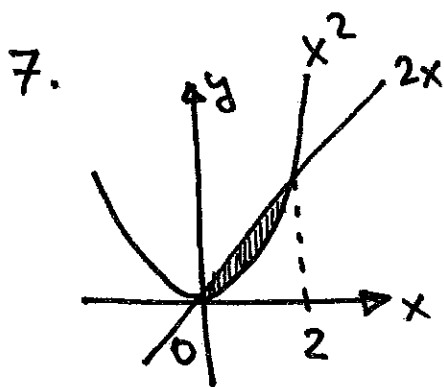
$$= \left(\ln 3 + \frac{1}{3} \right) - \left(\ln 1 + 1 \right) = \ln 3 - \frac{2}{3}$$

6.(a)



$$A = \int_{-2}^1 (e^x - (x+1)) dx$$

$$\begin{aligned}
 (b) \quad A &= \left(e^x - \frac{x^2}{2} - x \right) \Big|_{-2}^1 \\
 &= \left(e - \frac{1}{2} - 1 \right) - \left(e^{-2} - \cancel{2} + \cancel{2} \right) \\
 &= e - e^{-2} - \frac{3}{2} \approx \underline{\underline{1.08}}
 \end{aligned}$$

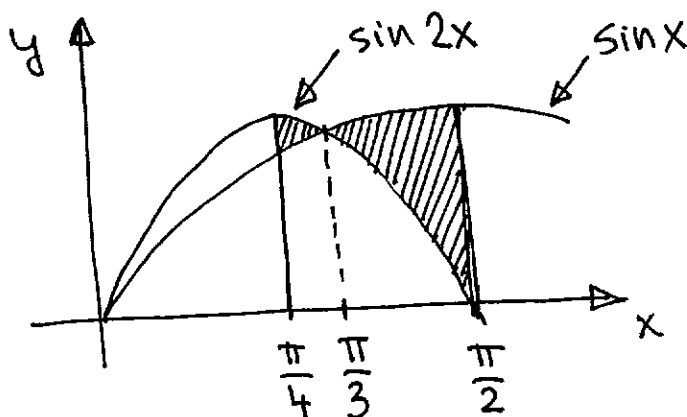


$$\begin{aligned}
 x^2 &= 2x \rightarrow x^2 - 2x = x(x-2) = 0 \\
 &\rightarrow x=0, 2 \text{ are} \\
 &\quad \text{pts. of intersection}
 \end{aligned}$$

$$A = \int_0^2 (2x - x^2) dx$$

$$= \left(x^2 - \frac{x^3}{3} \right) \Big|_0^2 = 4 - \frac{8}{3} = \underline{\underline{\frac{4}{3}}}$$

8. (a)



$$\sin 2x = \sin x$$

$$2\sin x \cos x = \sin x$$

$$\rightarrow \sin x (2\cos x - 1) = 0$$

$$\sin x = 0 \rightarrow x = 0, \pi, 2\pi, \dots$$

$$2\cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$$

(we need solutions between $\pi/4$ and $\pi/2$)

$$A = \int_{\pi/4}^{\pi/3} (\sin 2x - \sin x) dx + \int_{\pi/3}^{\pi/2} (\sin x - \sin 2x) dx$$

$$(b) = \left(-\frac{1}{2} \cos 2x + \cos x \right) \Big|_{\pi/4}^{\pi/3} + \left(-\cos x + \frac{1}{2} \cos 2x \right) \Big|_{\pi/3}^{\pi/2}$$

$$\begin{aligned} & \left. \begin{array}{l} \cos \frac{\pi}{3} = \frac{1}{2} \\ \cos \frac{2\pi}{3} = -\frac{1}{2} \end{array} \right| = \left(-\frac{1}{2} \cos \frac{2\pi}{3} + \cos \frac{\pi}{3} \right) - \left(-\frac{1}{2} \cos \frac{\pi}{2} + \cos \frac{\pi}{4} \right) \\ & \quad + \left(-\cos \frac{\pi}{2} + \frac{1}{2} \cos \pi \right) - \left(-\cos \frac{\pi}{3} + \frac{1}{2} \cos \frac{2\pi}{3} \right) \\ & = \left(-\frac{1}{2} \left(-\frac{1}{2} \right) + \frac{1}{2} \right) - \left(\frac{\sqrt{2}}{2} \right) \\ & \quad + \left(-\frac{1}{2} \right) - \left(-\frac{1}{2} + \frac{1}{2} \left(-\frac{1}{2} \right) \right) \\ & = \frac{3}{4} - \frac{\sqrt{2}}{2} - \frac{1}{2} + \frac{3}{4} = 1 - \frac{\sqrt{2}}{2} \end{aligned}$$

END