Assignment 24.5

1.(a) $x^3/x-3$ is not continuous at $x=3$

(b) $x^{-1/2}$ is not cont. at $x=0$

(c) $x^{-1/2}$ is not cont. at $x=0$ and upper limit is ∞

(d) upper limit is ∞

(e) $\dfrac{1}{x(x+3)}$ is not continuous at $x=0$ and $x=-3$

(f) x^{-3} is not continuous at $x=0$; integration limits are $-\infty$ and ∞

2. (a) $$\int_0^\infty \frac{1}{2x+4}\,dx = \lim_{T\to\infty}\int_0^T \frac{1}{2x+4}\,dx$$

$$= \lim_{T\to\infty} \frac{1}{2}\ln|2x+4|\Big|_0^T = \lim_{T\to\infty}\Big(\frac{1}{2}\underbrace{\ln|2T+4|}_{\infty} - \frac{1}{2}\ln|4|\Big)$$

$$= \infty \to \text{DIVERGES}$$

(b) $$\int_{-2}^0 \frac{1}{2x+4}\,dx = \lim_{T\to-2^+}\int_T^0 \frac{1}{2x+4}\,dx$$

$$= \lim_{T\to-2^+}\frac{1}{2}\ln|2x+4|\Big|_T^0 = \lim_{T\to-2^+}\Big(\frac{1}{2}\ln|4| - \frac{1}{2}\ln|2T+4|\Big)$$

$$= \frac{1}{2}\ln 4 - \frac{1}{2}\underbrace{\ln 0}_{-\infty} = \infty \to \text{DIVERGES}$$

(c) $$\int_{-2}^\infty \frac{1}{2x+4}\,dx = \int_{-2}^0 \frac{1}{2x+4}\,dx + \int_0^\infty \frac{1}{2x+4}\,dx$$

both integrals are DIVERGENT

3. (a) $\displaystyle\int_0^\infty e^{-10x}dx = \lim_{T\to\infty}\int_0^T e^{-10x}\,dx$

$\displaystyle = \lim_{T\to\infty}\left(-\frac{1}{10}\right)e^{-10x}\Big|_0^T = \lim_{T\to\infty}\Big(-\frac{1}{10}\cdot\underbrace{e^{-10T}}_{0}\Big)-\Big(-\frac{1}{10}e^{0}\Big)$

$\displaystyle = \frac{1}{10} \Rightarrow$ Convergent

(b) $\displaystyle\int_0^\infty x e^{-x^2}dx = \lim_{T\to\infty}\int_0^T x e^{-x^2}dx = \lim_{T\to\infty}\left(-\frac{1}{2}e^{-x^2}\right)\Big|_0^T$

int. by substitution

$\displaystyle = \lim_{T\to\infty}\Big(-\frac{1}{2}\underbrace{e^{-T^2}}_{0}\Big) - \Big(-\frac{1}{2}e^{0}\Big) = \frac{1}{2} \to$ convergent

(c) $\displaystyle\int_{-\infty}^1 2^x\,dx = \lim_{T\to-\infty}\int_T^1 2^x dx = \lim_{T\to-\infty}\frac{2^x}{\ln 2}\Big|_T^1$

$\displaystyle = \lim_{T\to-\infty}\Big(\frac{2}{\ln 2} - \frac{\overbrace{2^T}^{0}}{\ln 2}\Big) = \frac{2}{\ln 2} \to$ conv.

(d) $\displaystyle\int_{-\infty}^0 \frac{1}{1+x^2}dx = \lim_{T\to-\infty}\arctan x\Big|_T^0$

$\displaystyle = \lim_{T\to-\infty}(0-\arctan T) = -\underbrace{\arctan(-\infty)}_{-\pi/2} = \frac{\pi}{2} \Rightarrow$ conv.

(e) $\displaystyle\int_0^4 \frac{1}{x-4}dx = \lim_{T\to4^-}\ln|x-4|\Big|_0^T$

$\displaystyle = \lim_{T\to4^-}\ln|T-4| - \ln|-4| = \ln 0 - \ln 4 = -\infty \to$ div.

(f) $$\int_0^4 \frac{1}{(x-4)^2}\,dx = -\lim_{T\to 4^-} \left.\frac{1}{x-4}\right|_0^T$$

$$= -\lim_{T\to 4^-}\left(\frac{1}{T-4} - \underbrace{\frac{1}{0-4}}_{1/4}\right) = -(-\infty) = \infty$$