ASSIGNMENT 24

1. (a) $f(x) = \arctan x \;\dots\; f(0)=0$

$$f'(x) = \frac{1}{1+x^2} \;\dots\; f'(0)=1$$

$$f''(x) = (-1)(1+x^2)^{-2}(2x) = -\frac{2x}{(1+x^2)^2} \;\dots\; f''(0)=0$$

$$f'''(x) = \frac{-2(1+x^2)^2 - 2x\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4} \;\dots\; f'''(0)=-2$$

$$\text{So } T_3(x) = \underbrace{f(0)}_{0} + \underbrace{f'(0)}_{1}x + \frac{\overbrace{f''(0)}^{0}}{2}x^2 + \frac{\overbrace{f'''(0)}^{-2}}{6}x^3$$

$$= \underline{\underline{x - \frac{x^3}{3}}}$$

(b)

$$\int_0^1 \arctan x\,dx \approx \int_0^1 \left(x - \frac{x^3}{3}\right)dx$$

$$= \left(\frac{x^2}{2} - \frac{x^4}{12}\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \approx \underline{\underline{0.417}}$$

2. We use $e^x \approx 1 + x + \frac{x^2}{2}$

$$\int_{0.1}^{1} \frac{e^x}{x^2}\,dx \approx \int_{0.1}^{1} \frac{1+x+x^2/2}{x^2}\,dx$$

$$= \int_{0.1}^{1} \left(\frac{1}{x^2} + \frac{1}{x} + \frac{1}{2}\right)dx$$

$$= \left(-\frac{1}{x} + \ln|x| + \frac{1}{2}x\right)\Big|_{0.1}^{1}$$

$$= \left(-1 + \frac{1}{2}\right) - \left(-10 + \ln 0.1 + \frac{1}{2}\cdot 0.1\right)$$

$$= -0.5 + 10 - \ln 0.1 - 0.05 = 11.75$$

3. (a) $T_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

so $e^{-x^2/2} \approx T_3\left(-\frac{x^2}{2}\right) = 1 - \frac{x^2}{2} + \frac{(-x^2/2)^2}{2} + \frac{(-x^2/2)^3}{6}$

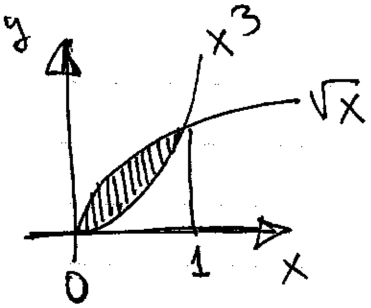
$$= 1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}$$

(b) $\int_0^1 e^{-x^2/2}\,dx \approx \int_0^1 \left(1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}\right) dx$

$$= \left(x - \frac{x^3}{6} + \frac{x^5}{40} - \frac{x^7}{336}\right)\Big|_0^1$$

$$= 1 - \frac{1}{6} + \frac{1}{40} - \frac{1}{336} \approx 0.855$$

4. (a)



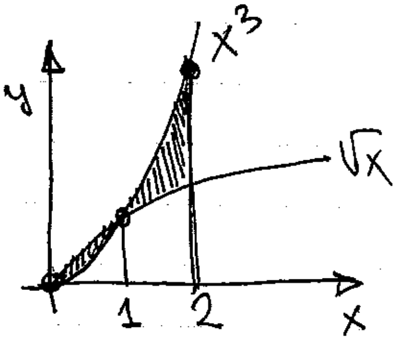
$A = \int_0^1 (\sqrt{x} - x^3)\,dx$

$= \frac{1}{3/2} x^{3/2} - \frac{x^4}{4}\Big|_0^1$

$= \frac{2}{3} x^{3/2} - \frac{x^4}{4}\Big|_0^1$

$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

(b)



$A = \int_0^1 (\sqrt{x} - x^3)\,dx$

$+ \int_1^2 (x^3 - \sqrt{x})\,dx$

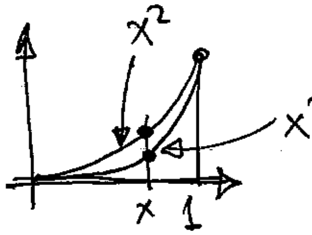
from (a) $\rightarrow = \frac{5}{12} + \left(\frac{x^4}{4} - \frac{2}{3}\underbrace{x^{3/2}}_{\sqrt{x^3}}\right)\Big|_1^2$

$= \frac{5}{12} + \left(4 - \frac{2}{3}\sqrt{8}\right) - \left(\frac{1}{4} - \frac{2}{3}\right)$

$= \frac{5}{12} + \frac{53}{12} - \frac{2}{3}\sqrt{8} = \frac{58}{12} - \frac{2}{3}\sqrt{8} \approx 2.95$

5. (a) $\bar{f} = \frac{1}{1-0}\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$

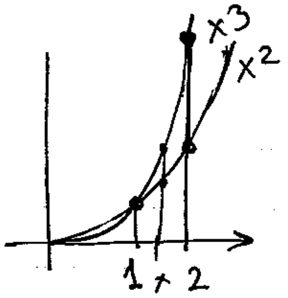
(b) $\bar{f} = \frac{1}{1-0}\int_0^1 x^3\,dx = \frac{x^4}{4}\Big|_0^1 = \frac{1}{4}$ $\qquad \frac{1}{3} > \frac{1}{4}$

(c) 

values of x^2 are larger than corresponding values of x^3
$\Rightarrow$ average of x^2 > average of x^3

6. (a) $\bar{f} = \frac{1}{2-1}\int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{7}{3}$

(b) $\bar{f} = \frac{1}{2-1}\int_1^2 x^3\,dx = \frac{x^4}{4}\Big|_1^2 = \frac{15}{4}$ $\qquad \frac{15}{4} > \frac{7}{3}$

(c) 

the values of x^3 are larger than the values of x^2
$\Rightarrow$ average of x^3 > average of x^2

7. (a) $\displaystyle\int_0^\infty \frac{1}{(1+2x)^{3/2}}\,dx = \lim_{T\to\infty}\int_0^T (1+2x)^{-3/2}\,dx$

$= \lim_{T\to\infty}\left(-\frac{1}{\sqrt{1+2x}}\right)\Big|_0^T$

$= \lim_{T\to\infty}\left(-\frac{1}{\sqrt{1+2T}}\right) - (-1)$

$= -\frac{1}{\infty} + 1 = \underline{\underline{1}}$

$\int (1+2x)^{-3/2}\,dx$

$= \left[\begin{array}{l} u = 1+2x \\ du/dx = 2 \to dx = du/2 \end{array}\right]$

$= \int u^{-3/2}\frac{du}{2} = \frac{u^{-1/2}}{-1/2}\cdot 2$

$= -u^{-1/2} = -\frac{1}{\sqrt{u}}$

$= -\frac{1}{\sqrt{1+2x}}$

(b) $$\int_{10}^{\infty} \frac{1}{x^2}\,dx = \lim_{T\to\infty} \int_{10}^{T} x^{-2}\,dx$$

$$= \lim_{T\to\infty} \left(-\frac{1}{x}\right)\Big|_{10}^{T} = \lim_{T\to\infty} \left(\underbrace{-\frac{1}{T}}_{0}\right) - \left(-\frac{1}{10}\right) = \underline{\underline{\frac{1}{10}}}$$

(c) $$\int_{1}^{\infty} e^{-0.5x}\,dx = \lim_{T\to\infty} \int_{1}^{T} e^{-0.5x}\,dx$$

$$= \lim_{T\to\infty} \underbrace{\frac{1}{-0.5}}_{=-2} e^{-0.5x} \Big|_{1}^{T}$$

$$= \lim_{T\to\infty} \left(-2e^{-0.5T}\right) - \left(-2e^{-0.5}\right)$$

$$= \underbrace{-2e^{-\infty}}_{0} + 2e^{-0.5} \approx \underline{\underline{1.21}}$$

8. (a) $$\text{avg density} = \frac{\text{total number}}{\underbrace{\text{total distance}}_{100\,\text{km}}} = \frac{2720}{100} = 27.2$$ monkeys/km

2720 ← from (b)

part (b): $$\text{total number} = \int_{0}^{100} 0.003x(248-x)\,dx$$

$$= 0.003 \int_{0}^{100} (248x - x^2)\,dx$$

$$= 0.003 \cdot \left(248\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{0}^{100}$$

$$= 0.003 \cdot \left(248 \cdot 5000 - \frac{1000000}{3}\right) = 2720$$ monkeys