ASSIGNMENT 29

1. I... $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx = 11$ ✓

(F) II... integral of product $\neq$ product of integrals

III... $\int_a^b (2f(x)-g(x))\,dx = \int_a^b 2f(x)\,dx - \int_a^b g(x)\,dx$

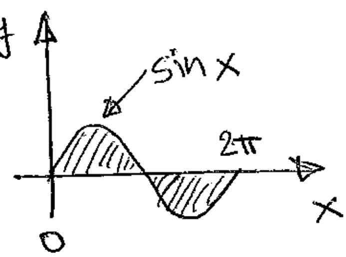
$$= 2\underbrace{\int_a^b f(x)\,dx}_{4} - \underbrace{\int_a^b g(x)\,dx}_{7} = 1$$ ✓

2. FALSE; counterexample:

$$\int \underbrace{2t}_{q(t)}\,dt = \underbrace{t^2}_{p(t)} \quad \text{but } q'(t) = (2t)' = 2 \neq p(t)$$

3. FALSE; $\left(\frac{1}{x}+C\right)' = -\frac{1}{x^2} \neq \ln x$

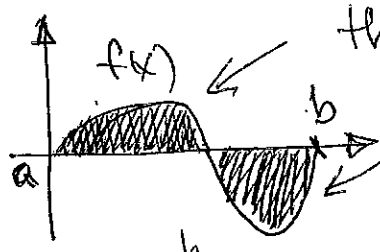
4. FALSE;



$$\int_0^{2\pi} \sin x\,dx = 0$$

but $\sin x \neq 0$

or:



these two regions have equal areas

then $\int_a^b f(x)\,dx = 0$ but $f(x) \neq 0$

5. $\int_1^2 f'(x)\,dx \overset{\text{FTC}}{=} f(x)\Big|_1^2 = f(2) - f(1) = 1$

TRUE

6. TRUE; since the definite integral is a real number

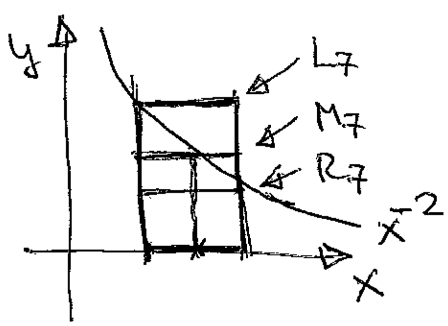
7. FALSE; incorrect use of FTC; should be

$$\int_{-1}^{1} x^4\,dx = \frac{x^5}{5}\Big|_{-1}^{1} = \ldots$$

antiderivative!

8. TRUE, by the def. of antiderivative

9.



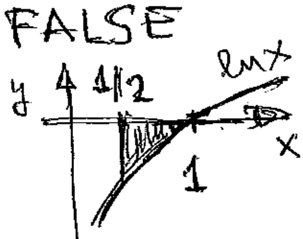
TRUE
(for any decreasing function)

10. FALSE integral of product $\neq$ product of integrals

or: differentiate $\frac{x^3}{3} \cdot \frac{e^{2x}}{2} = \frac{1}{6} x^3 e^{2x}$

and show that $\neq x^2 e^{2x}$

11. FALSE



$\ln x \le 0$ on $[1/2, 1]$

so $\int_{1/2}^{1} \ln x < 0$