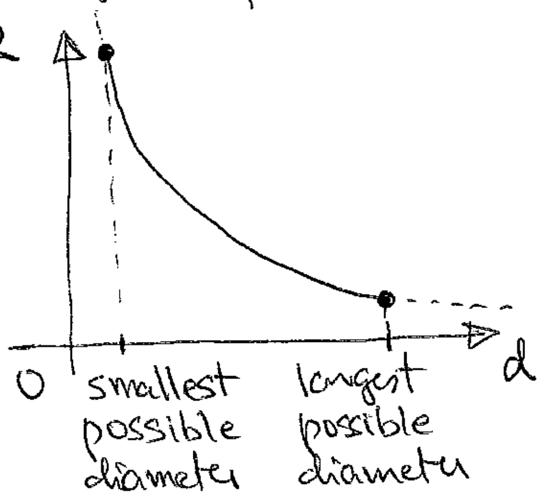
ASSIGNMENT 30

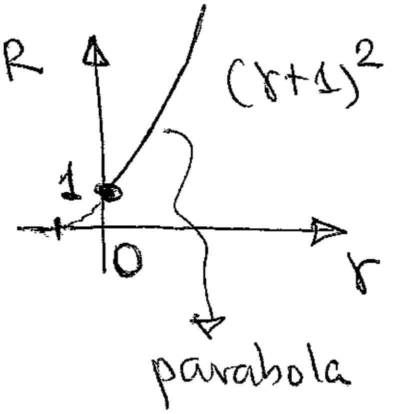
1. (a) R is proportional to viscosity K
proportional to the length l of the vessel
inversely proportional to the fourth power of the diameter d

(b) R is of the form $R = \frac{\text{constant}}{d^4}$ (so looks like $y = 1/x^4$)

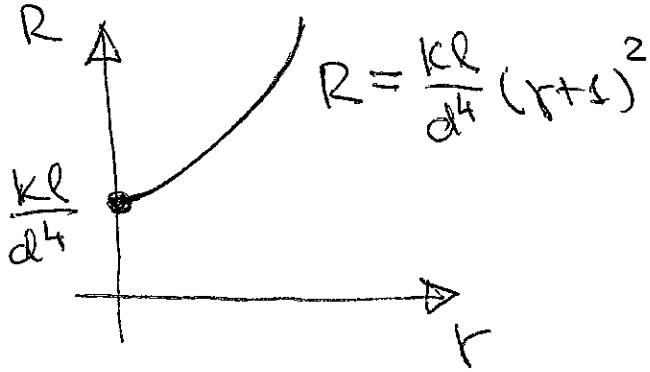


(c) Yes, as the diameter increases, the resistance of the flow decreases. Or: as a blood vessel shrinks the resistance increases.

(d) R is of the form $R = \underbrace{\text{constant}}_{= Kl/d^4} \cdot (r+1)^2$



If $\frac{Kl}{d^4} < 1$ ... compress
$\frac{Kl}{d^4} > 1$ ... expand
vertically



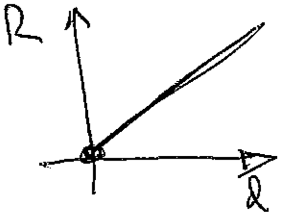
(e) Yes. As the vessel is curved more and more, the resistance increases

small curvature
small resistance

large curvature
large resistance

(f) $R = \frac{Kl(\gamma+1)^2}{d^4} = \frac{K(\gamma+1)^2}{d^4} \cdot l$

line of slope $K(\gamma+1)^2/d^4$ through the origin in lR-coordinate system

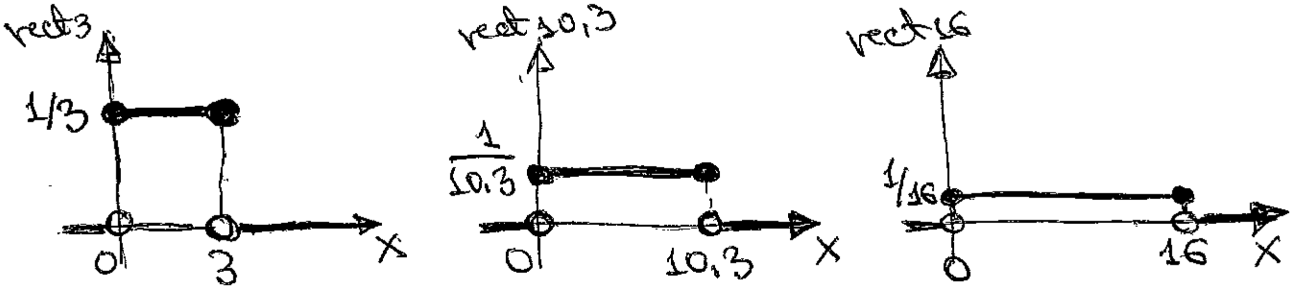


(g) $R = \frac{Kl(\gamma+1)^2}{d^4} = \frac{l(\gamma+1)^2}{d^4} \cdot K$

line of slope $\frac{l(\gamma+1)^2}{d^4}$ through the origin in KR-coordinate system.

2.(a) $rect_\ell$ is a piecewise defined function: between 0 and ℓ (including 0 and ℓ) it is a straight line of height $1/\ell$; otherwise, i.e. when $x<0$ and $x>0$ it lies on the x-axis

(b)



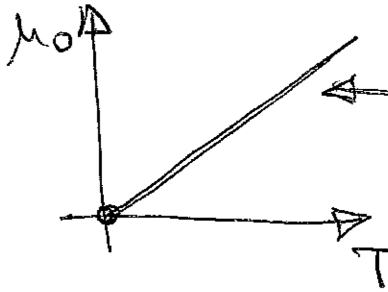
(c) The non-zero horizontal piece of $rect_\ell$ becomes longer and longer, but comes closer and closer to the x-axis.

3. (a)
$$\mu_0 = \underbrace{\frac{\sqrt{3}\,k_B T}{4 p l_m x_0}\left(\frac{x_0}{2(1-x_0)^3} - \frac{1}{4(1-x_0)^2} + \frac{1}{4}\right)}_{\text{slope}} + \underbrace{\frac{\sqrt{3}\,k_p (n+1)}{4\,\ell_0^{m+1}}}_{\text{vertical intercept}}$$

line!

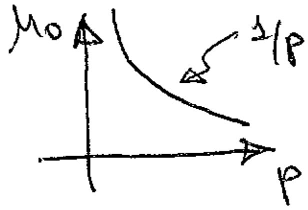
(b)
$$\mu_0 = \frac{\sqrt{3}\,k_B T}{4 p l_m \cdot \frac{1}{2}}\left(\underbrace{\frac{\frac{1}{2}}{\frac{1}{4}} - \frac{1}{1} + \frac{1}{4}}_{5/4}\right) + 0$$

$$\mu_0 = \frac{\sqrt{3}\,k_B \cdot 5}{8 p l_m} \cdot T$$

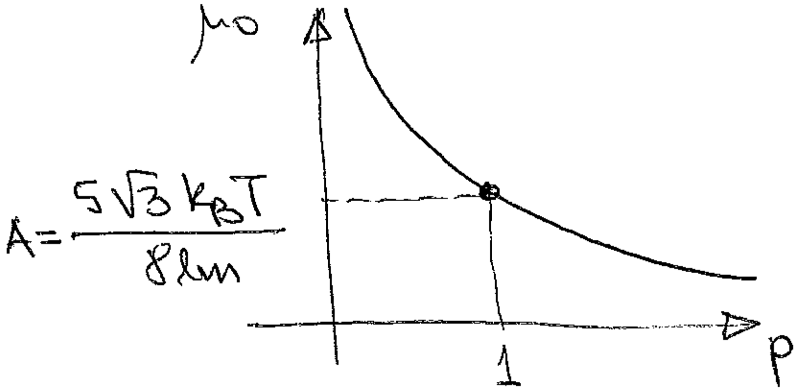


line of slope

(c) Rewrite μ_0 from (b) as

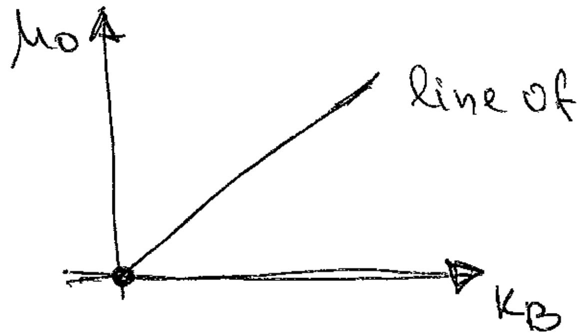


$$\mu_0 = \underbrace{\frac{5\sqrt{3}\,k_B T}{8\,l m}}_{\text{this is constant, call it A}} \cdot \frac{1}{p}$$



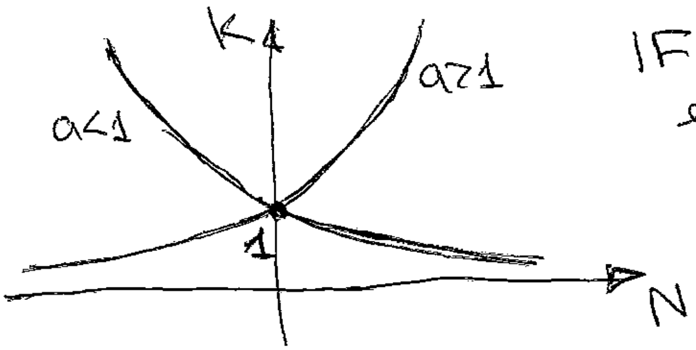
if $A > 1$ expand $\frac{1}{p}$
if $A < 1$ compress $\frac{1}{p}$
vertically by a factor of A

(d) Rewrite μ_0 from (b) as $\mu_0 = \dfrac{5\sqrt{3}\,T}{8\,p\,l m}\, k_B$



line of (positive) slope $\dfrac{5\sqrt{3}\,T}{8\,p\,l m}$

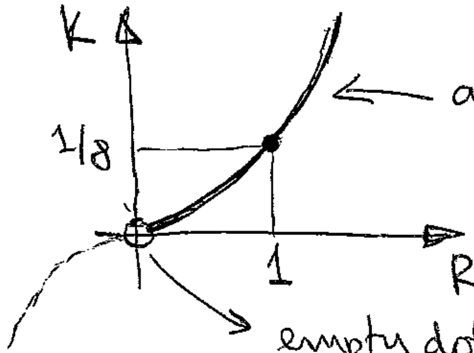
4(a) $K \approx \left(\dfrac{5R}{\sigma_E}\right)^N = a^N$ where $a = \dfrac{5R}{\sigma_E} > 0$



IF $a > 1$ then it is exp. increasing

IF $a<1$ ... exp. decreasing
it could happen that $a=1$; then the graph of K is a horizontal line with intercept 1

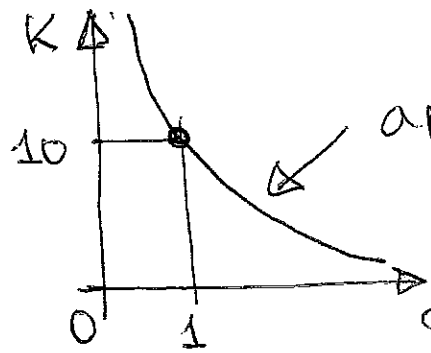
(b) $K \approx \left(\frac{5R}{10}\right)^3 = \left(\frac{R}{2}\right)^3 = \frac{1}{8}R^3$ ... compressed graph of R^3



approximate graph of K

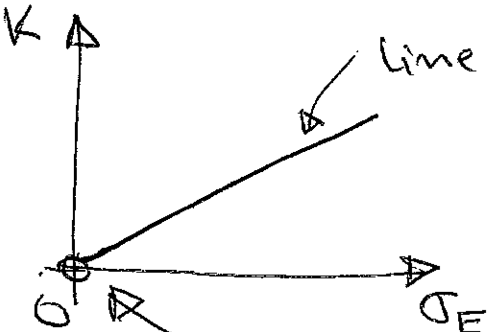
empty dot because it is given that $R>0$

(c) $K \approx \left(\frac{5.2}{\sigma_E}\right)^1 = 10 \cdot \frac{1}{\sigma_E}$ ... vertically expanded graph of $1/\sigma_E$



approximate graph of K

(d) $K \approx \left(\frac{5.2}{\sigma_E}\right)^{-1} = \frac{1}{10}\sigma_E$



line of slope 1/10

empty dot because $\sigma_E > 0$

(e) $K \approx \left(\dfrac{5R}{\sigma_E} \right)^2$

$$\underset{K(R)}{\overset{K}{\shortparallel}}$$

$$\rightarrow K(3R) \approx \left(\frac{5 \cdot 3R}{\sigma_E} \right)^2 = 3^2 \cdot \left(\frac{5R}{\sigma_E} \right)^2 = 3^2 \cdot K(R)$$

increases (approximately) nine-fold

(f) $K(R) \approx \left(\dfrac{5R}{\sigma_E} \right)^{-2}$

$$\rightarrow K(3R) \approx \left(\frac{5 \cdot 3R}{\sigma_E} \right)^{-2} = 3^{-2} \left(\frac{5R}{\sigma_E} \right)^{-2} = \frac{1}{9} \cdot K(R)$$

decreases (approx.) 9-fold

(g) $K(\sigma_E) \approx \left(\dfrac{5R}{\sigma_E} \right)^3$

$$\rightarrow K(2\sigma_E) \approx \left(\frac{5R}{2\sigma_E} \right)^3 = \left(\frac{1}{2} \right)^3 \left(\frac{5R}{\sigma_E} \right)^3 = \frac{1}{8} K(\sigma_E)$$

decreases (approx.) 8-fold

(h) $K(\sigma_E) \approx \left(\dfrac{5R}{\sigma_E} \right)^{-1}$

$$\rightarrow K(2\sigma_E) \approx \left(\frac{5R}{2\sigma_E} \right)^{-1} = \left(\frac{1}{2} \right)^{-1} \left(\frac{5R}{\sigma_E} \right)^{-1} = 2\, K(\sigma_E)$$

increases (approx.) 2-fold

i.e. (approx.) doubles