

ASSIGNMENT 31

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$$1. (a) \quad \sinh 0.2 = \frac{e^{0.2} - e^{-0.2}}{2} \approx 0.20134$$

$$\sinh 3 = \frac{e^3 - e^{-3}}{2} \approx 10.01787$$

$$(b) \quad p = 0.267(10) + \frac{1.3}{0.5} \cdot \frac{\sinh 0.2}{\sinh 3}$$

$$\approx 2.67 + 2.6 \cdot \frac{0.20134}{10.01787} \approx 2.72226$$

$$(c) \quad p = 0.267p_i + \frac{1.3}{r} \cdot \frac{(e^{0.4r} - e^{-0.4r})/2}{(e^3 - e^{-3})/2}$$

← cancel by 1/2

$$= 0.267p_i + \frac{1.3}{r} \cdot \frac{e^{0.4r} - e^{-0.4r}}{e^3 - e^{-3}}$$

2. (a) $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are functions of t
parameters are: $\hat{v}_0, a, f_m, \hat{d}, \hat{d}_{lag}$

$$(b) \quad \cos(2\pi f_m(t + \hat{d}))$$

↖ $\hat{d}$ is a shift, so it does not affect the period

$$\text{period} = \frac{2\pi}{2\pi f_m} = \frac{1}{f_m}$$

(c) same as (b)

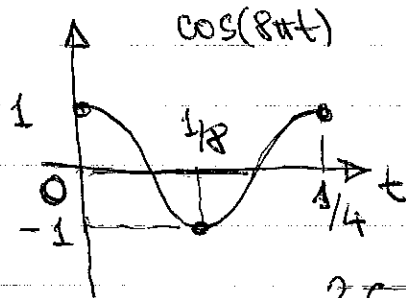
$$(d) \quad \hat{\lambda}_1 = 0 + 2.6 \cos(2\pi \cdot 4(t + \pi))$$

↙ amplitude = 2.6 ↓ period = 1/4 ↘ phase = -π

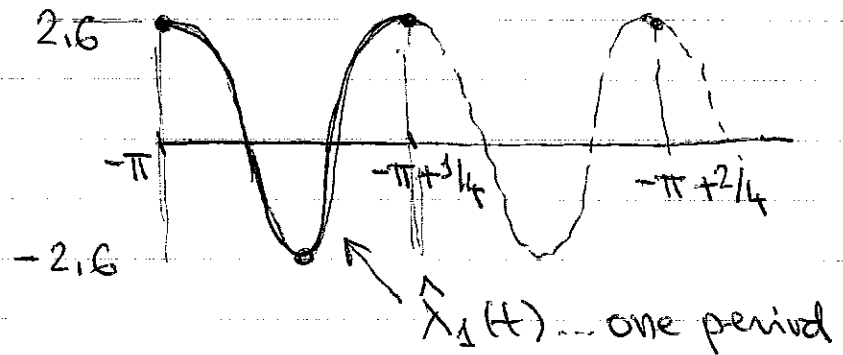
to graph, consider the sequence of transformations:

$$\cos t \rightarrow \cos(8\pi t) \rightarrow \cos(8\pi(t+\pi)) \rightarrow 2.6 \cos(8\pi(t+\pi))$$

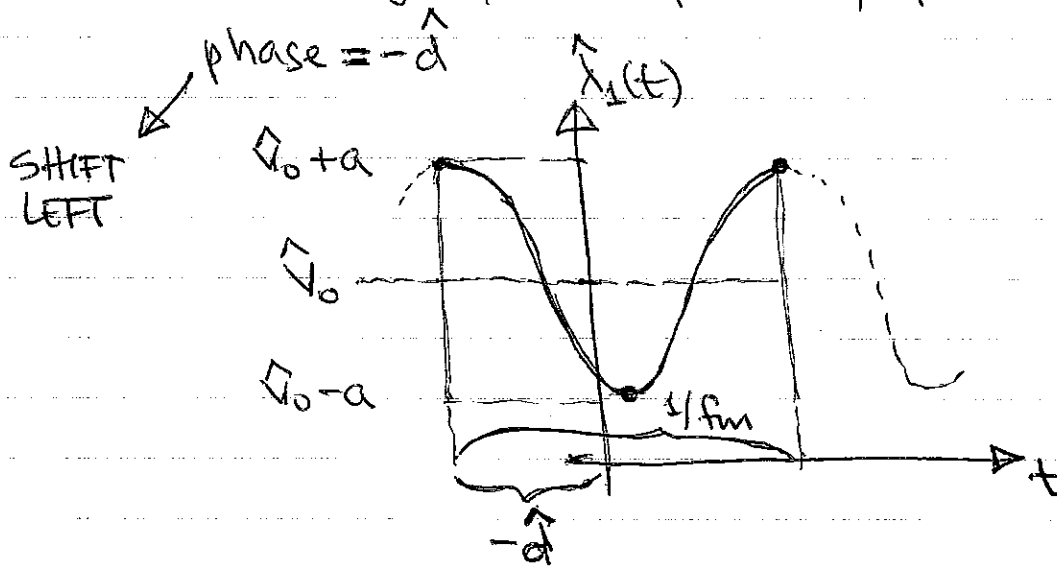
main period:



shift left by π and expand vertically by a factor of 2.6:



(e) $\hat{V}_0 = \text{average}$, $a = \text{amplitude}$, $\text{period} = \frac{2\pi}{2\pi f_m} = \frac{1}{f_m}$



(f) shift $\hat{\lambda}_1(t)$ to the right by the amount equal to $|\hat{d}_{\text{lag}}|$ absolute value of $\hat{d}_{\text{lag}}$

3.(a) first peak: $t=21$; second peak: $t=44 \rightarrow$ period = 23
 (check: third peak should be at $44+23=67$;
 looks like it is)

(b) minimum: 0.35; maximum: 1.8
 average = $(1.8+0.35)/2 = 1.075$
 amplitude = $1.8 - 1.075 = 0.725$

(c) shift right = 21

$$\text{so } B_{\text{mal}}(t) \approx 1.075 + 0.725 \cos\left(\frac{2\pi}{23}(t-21)\right)$$

(d) first peak: $t=8$; second peak: $t=32 \rightarrow$ period = 24
 (check: third peak: $32+24=56$)

minimum: 0.8, maximum: 1.2
 average = 1, amplitude = 0.2

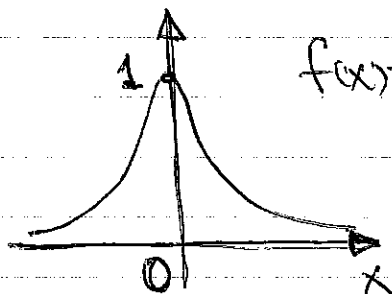
shift right = 8

$$REV-ERB_N(t) = 1 + 0.2 \cos\left(\frac{2\pi}{24}(t-8)\right)$$

(e) amplitude = 0.1
 shift right = 18

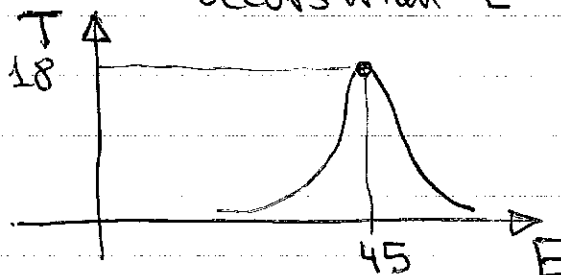
$$ROR_N = 1 + 0.1 \cos\left(\frac{2\pi}{24}(t-18)\right)$$

4. (a)

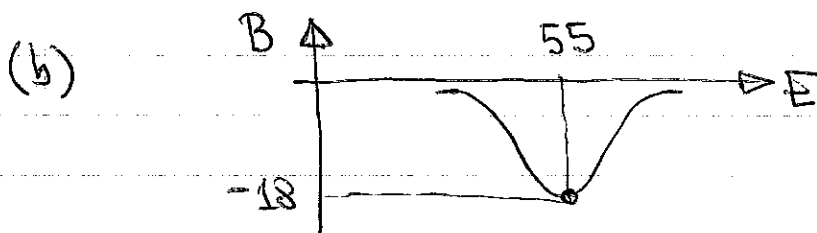


$$f(x) = \frac{1}{1+x^2}$$

top curve: maximum = 18
occurs when $E = 45$



$$\text{so } T(E) = 18 \cdot \frac{1}{1+(E-45)^2}$$



$$B(E) = -18 \cdot \frac{1}{1+(E-55)^2}$$

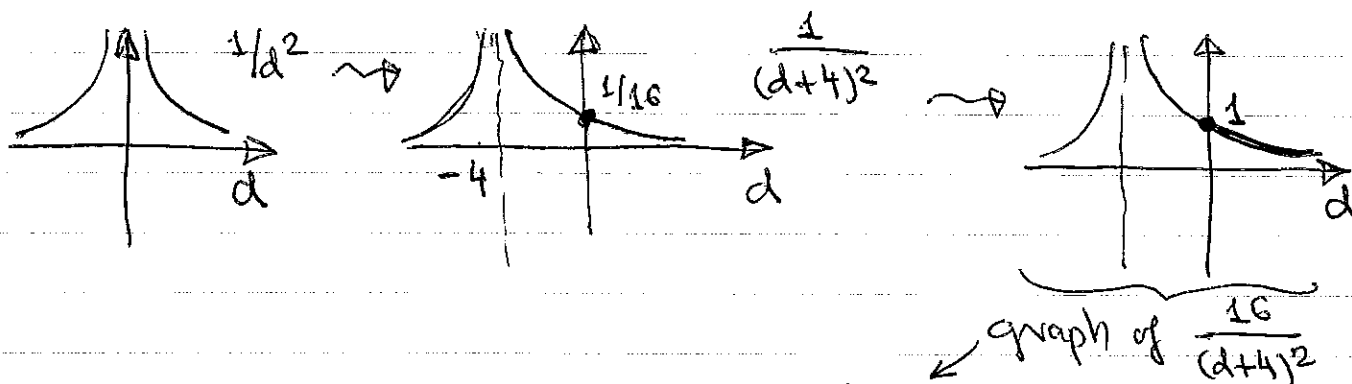
(NOTE: 18, 45 and 55 are estimates)

5. (a) $f(0) = \pi$; $f(5) = \frac{1}{(1+5/4)^2}$

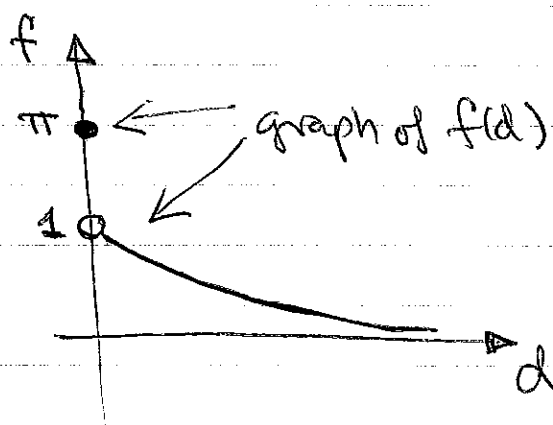
(b) $f(d) = \begin{cases} \pi & \text{if } d=0 \\ \frac{1}{(1+d/4)^2} & \text{if } d>0 \end{cases}$

$$= \frac{1}{\left(\frac{1}{4}(4+d)\right)^2} = \frac{1}{\frac{1}{16}(4+d)^2} = 16 \cdot \frac{1}{(d+4)^2}$$

SIMPLIFY TO SEE HOW TO TRANSFORM



we need only $d > 0$ part



ESTIMATE

6. (a) GIVEN when $\log r = -5$, $\log h = 0.1$; when $\log r = 1$, $\log h = 2.4$ (ESTIMATE)

so slope = $\frac{2.4 - 0.1}{1 - (-5)} = \frac{2.3}{6} \approx 0.38$

point-slope equation:

$$\log h - 0.1 = 0.38(\log r - (-5))$$

$$\log h = 0.38 \log r + 2$$

(b) $10^{\log h} = 10^{0.38 \log r + 2}$

$$h = 10^{\log r \cdot 0.38} \cdot 10^2$$

$$h = 100 \cdot r^{0.38}$$